1. Continuity and Differentiability 2. Differentiation

Course: **JEE Main, GUJCET** Class: 12 Subject: **MATHS**

Continuity and Differentiability

LEVEL 1 (*NCERT Based*)

1. If the function $f(x)=\begin{cases}\dfrac{\sqrt{2+\cos x}-1}{(\pi-x)^2}, & x\neq\pi\\ k, & x=\pi\end{cases}$ is continuous at $x=\pi$, then k equals:

(A)0 (B) 1/2 (C) 2 (D) 1/4 **[JEE Main Online 2014]**

2. Let $f(x)=\dfrac{1-\tan x}{4x-\pi}, x\neq\dfrac{\pi}{4}, x\in\left[0,\dfrac{\pi}{2}\right]$. If $f(x)$ is continuous in $\left[0,\dfrac{\pi}{2}\right]$,then $f\left(\dfrac{\pi}{4}\right)$ is

(A) 1 (B) 1/2 (C) - 1/2 (D) -1 **[AIEEE 2004]**

LEVEL 2 (*NCERT* plus)

3. Function $f:[1.2,1.9]\to R, f(x)=[x]$, where $[x]$ denotes the greatest integer less than or equal to x. Then _____

(A) $f'(x)=1$ (B) f is not differentiatitive (C) $f'(x)=0$ (D) f is not continuous funtion

[GUJCET 2023]

4. If the function $f(x)=\begin{cases}-x, & x<1\\ a+\cos^{-1}(x+b), & 1\le x\le 2\end{cases}$ is differentiable at $x=1$,then $\dfrac{a}{b}$ is equal to:

(A) $-1-\cos^{-1}(2)$ (B) $\dfrac{\pi-2}{2}$ (C) $\dfrac{-\pi-2}{2}$ (D) $\dfrac{\pi+2}{2}$

[JEE Main Online 2016]

5. If the function $g(x)=\begin{cases}k\sqrt{x+1}, & 0\le x\le 3\\ mx+2, & 3<x\le 5\end{cases}$ is differentiable, then the value of $k+m$ is :

(A) $\dfrac{16}{5}$ (B) $\dfrac{10}{3}$ (C) 4 (D) 2 **[JEE Main 2015]**

6. The value of p and q for which the function $f(x)=\begin{cases}\dfrac{\sin(p+1)x+\sin x}{x}, & x<0\\ q, & x=0\\ \dfrac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}, & x>0\end{cases}$ is continuous for all x in R, are:

(A) $p=\dfrac{1}{2}, q=-\dfrac{3}{2}$ (B) $p=\dfrac{5}{2}, q=\dfrac{1}{2}$ (C) $p=-\dfrac{3}{2}, q=\dfrac{1}{2}$ (D) $p=\dfrac{1}{2}, q=\dfrac{3}{2}$ **[AIEEE 2011]**

7. The set of points where $f(x)=\frac{x}{1+|x|}$ is differentiable is

(A) $(0,\infty)$ (B) $(-\infty,0)\cup(0,\infty)$ (C) $(-\infty,-1)\cup(-1,\infty)$ (D) $(-\infty,\infty)$ **[AIEEE 2006]**

LEVEL 3 (*Non NCERT Based*)

8. Let $f(x)=\begin{cases}(x-1)^{\frac{1}{2-x}}, & x>1,\ x\neq 2\\ k, & x=2\end{cases}$. The value of k for which f is continuous at $x=2$ is:

(A) 1 (B) e (C) e^{-1} (D) e^{-2} **[JEE Main Online 2018]**

9. If the function f defined as $f(x)=\frac{1}{x}-\frac{k-1}{e^{2x}-1}, x\neq 0$, is continuous at $x=0$, then the ordered pair $(k, f(0))$ is equal to :

(A) $(3,2)$ (B) $(3,1)$ (C) $(2,1)$ (D) $\left(\frac{1}{3},2\right)$ **[JEE Main Online 2018]**

10. The value of k for which the function $f(x)=\begin{cases}\left(\frac{4}{5}\right)^{\frac{\tan 4x}{\tan 5x}}, & 0<x<\frac{\pi}{2}\\ k+\frac{2}{5}, & x=\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$, is :

(A) $\frac{17}{20}$ (B) $\frac{3}{5}$ (C) $-\frac{2}{5}$ (D) $\frac{2}{5}$ **[JEE Main Online 2017]**

11. For $x\in R, f(x)=|\log 2-\sin x|$ and $g(x)=f(f(x))$, then :

(A) g is not differentiable at $x=0$ (B) $g'(0)=\cos(\log 2)$

(C) $g'(0)=-\cos(\log 2)$ (D) g is differentiable at $x=0$ and $g'(0)=-\sin(\log 2)$

[JEE Main 2016]

12. Let $a,b\in R,(a\neq 0)$. If the function f defined as

$$f(x)=\begin{cases}\frac{2x^2}{a} & ,0\le x<1\\ a & ,1\le x<\sqrt{2}\\ \frac{2b^2-4b}{x^3} & ,\sqrt{2}\le x<\infty\end{cases}$$

is continuous in the interval $[0,\infty)$, then an ordered pair (a,b) is:

(A) $(-\sqrt{2},1+\sqrt{3})$ (B) $(\sqrt{2},1-\sqrt{3})$ (C) $(\sqrt{2},-1+\sqrt{3})$ (D) $(-\sqrt{2},1-\sqrt{3})$ **[JEE Main 2016]**

13. Let $f(x)$ be a polynomial of degree four having extreme values at $x=1$ and $x=2$. If $\lim\limits_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]=3$, then $f(2)$ is equal to :

(A) -4 (B) 0 (C) 4 (D) -8 **[JEE Main 2015]**

14. Let k be a non-zero real number. If $f(x)=\begin{cases}\dfrac{\left(e^x-1\right)^2}{\sin\left(\frac{x}{k}\right)\log\left(1+\frac{x}{4}\right)}, & x\neq 0\\ 12, & x=0\end{cases}$ is a continuous function,then the value of k is :

(A)1 (B) 2 (C) 3 (D) 4 **[JEE Main Online 2015]**

15. Let $f: R\to R$ be a function such that $|f(x)|\le x^2$, for all $x\in R$. Then at $x=0$, f is:

(A)continuous but not differentiable. (B)continuous as well as differentiable.
(C) neither continuous nor differentiable. (D)differentiable but not continuous **[JEE Main Online 2014]**

16. Let $f, g: R\to R$ be two functions defined by $f(x)=\begin{cases}x\sin\left(\frac{1}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$ and $g(x)=xf(x)$.

Statement I: f is continuous function at $x=0$.

Statement II: g is differentiable function at $x=0$.

(A) Both statements I and II are false. (B) Both statements I and II are true.
(C) Statement-I is true; Statement-II is false. (D) Statement-I is false; Statement-II is true.
[JEE Main Online 2014]

17. Let $f(x)=x|x|$, $g(x)=\sin x$ and $h(x)=(gof)(x)$. Then

(A) $h(x)$ is not differentiable at $x=0$.

(B) $h(x)$ is differentiable at $x=0$, but $h'(x)$ is not continuous at $x=0$.

(C) $h'(x)$ is continuous at $x=0$ but it is not differentiable at $x=0$.

(D) $h'(x)$ is differentiable at $x=0$. **[JEE Main Online 2014]**

18. If $f(x)$ is continuous and $f(9/2)=2/9$, then $\lim\limits_{x\to 0} f\left(\frac{1-\cos 3x}{x^2}\right)$ is equal to:

(A) 9/2 (B) 2/9 (C) 0 (D) 8/9 **[JEE Main Online 2014]**

19. Let f be a composite function of x defined by $f(u)=\frac{1}{u^2+u-2}$, $u(x)=\frac{1}{x-1}$.Then the number of points x where f is discontinuous is:

(A) 4 (B) 3 (C)2 (D) 1 **[JEE Main Online 2013]**

20. Let $f(x)=-1+|x-2|$, and $g(x)=1-|x|$; then the set of all points where fog is discontinuous is:

(A) $\{0,2\}$ (B) $\{0,1,2\}$ (C) $\{0\}$ (D)an empty set
[JEE Main Online 2013]

21. Consider the function: $f(x)=[x]+|x-1|,-1\le x\le 3$, where $[x]$ is the greatest integer function.

Statement I: f is not continuous at $x=0,1,2$ and 3.

Statement II: $f(x)=\begin{cases}-x, & -1\le x<0\\ 1-x, & 0\le x<1\\ 1+x, & 1\le x<2\\ 2+x, & 2\le x\le 3\end{cases}$

(A) Statement-1 is true; Statement-2 is false.
(B) Statement-1 is true; Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.
(C) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(D) Statement-1 is false; Statement-2 is true. **[JEE Main Online 2013]**

22. Consider the function $f(x)=|x-2|+|x-5|,\ x\in R$

Statement 1 : $f'(4)=0$

Statement 2 : f is continuous in [2,5] differentiable in (2,5) and $f(2)=f(5)$.

(A)Statement 1 is true, statement 2 is true ; statement 2 is NOT a correct explanation for Statement 1
(B)Statement 1 is true, statement 2 is false
(C)Statement 1 is false statement 2 is true
(D)Statement 1 is true, statement 2 is true ; statement 2 is a correct explanation for Statement 1
[AIEEE 2012]

23. Let $f(x)=\begin{cases}x^2\left|\cos\dfrac{\pi}{x}\right|, & x\neq 0\\ 0, & x=0\end{cases}$ then f is

(A) Diferentiable both at $x=0$ and at $x=2$
(B) Differentiable at $x=0$ but not differentiable at $x=2$
(C) Not differentiable at $x=0$ but differentiable at $x=2$
(D) Differentiable neither at $x=0$ nor at $x=2$
[JEE 2012 Advanced; Paper 1: Single Correct Answer Type]

24. If $f: R\to R$ is a function defined by $f(x)=[x]\cos\left(\dfrac{2x-1}{2}\right)\pi$, where $[x]$ denotes the greatest integer function, then f is :

(A) discontinuous only at non-zero integral values of x .(B) continuous only at $x=0$
(C) continuous for every real x. (D) discontinuous only at $x=0$ **[AIEEE 2012]**

25. If function $f(x)$ is differentiable at $x=a$, then $\lim\limits_{x\to a}\dfrac{x^2f(a)-a^2f(x)}{x-a}$ is :

(A) $-a^2f'(a)$ (B) $af(a)-a^2f'(a)$ (C) $2af(a)-a^2f'(a)$ (D) $2af(a)+a^2f'(a)$
[AIEEE 2011 Retest]

26. If $f(x)=\begin{cases} -x-\frac{\pi}{2}, & x\le -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2}<x\le 0 \\ x-1, & 0<x\le 1 \\ \ln x, & x>1 \end{cases}$, then

(A) $f(x)$ is continuous at $x=-\frac{\pi}{2}$ (B) $f(x)$ is not differentiable at $x=0$

(C) $f(x)$ is differentiable at $x=1$ (D) $f(x)$ is differentiable at $x=-\frac{3}{2}$

[JEE Advanced 2011; Paper 2: Multiple Correct Answer Type]

27. Define $F(x)$ as the product of two real functions $f_1(x)=x, x\in R$ and $f_2(x)=\begin{cases} x\sin\frac{1}{x}, & if\ x\neq 0 \\ 0, & if\ x=0 \end{cases}$

as follows: $F(x)=\begin{cases} f_1(x).f_2(x), & \text{if } x\neq 0 \\ 0, & \text{if } x=0 \end{cases}$

Statement 1: $F(x)$ is continuous on R.

Statement 2: $f_1(x)$ and $f_2(x)$ are continuous on R.

(A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true **[AIEEE 2011 Retest]**

28. Let $f(x)=x|x|$ and $g(x)=\sin x$

Statement 1: gof is differentiable at x = 0 and its derivative is continuous at that point.
Statement 2: gof is twice differentiable at x = 0.
(A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true **[AIEEE 2009]**

29. Let $f(x)=\begin{cases} (x-1)\sin\frac{1}{x-1}, & x=0 \\ 0, & if\ x=1 \end{cases}$, then which one of the following is true ?

(A) f is neither differentiable at $x=0$ nor at $x=1$ (B) f is differentiable at $x=0$ and at $x=1$.
(C) f is differentiable at $x=0$ but not at $x=1$. (D) f is differentiable at $x=1$ but not at $x=0$.
[AIEEE 2008]

30. Let $f : R \to R$ be a function defined by $f(x) = Min\{x+1, |x|+1\}$. Then which of the following is true ?

(A) $f(x) \geq 1$ *for all* $x \in R$ (B) $f(x)$ *is not differentiable at* $x = 1$.

(C) $f(x)$ *is differentiable everywhere.* (D) $f(x)$ *is not differentiable at* $x = 0$. **[AIEEE 2007]**

31. The function $f : R \sim \{0\} \to R$ given by $f(x) = \frac{1}{x} - \frac{2}{e^{2x}-1}$ can be made continuous at $x = 0$ by defining $f(0)$ as

(A) 2 (B) -1 (C) 0 (D) 1 **[AIEEE 2007]**

32. Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$, and

$$\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1 \text{ for each } x > 0. \quad \text{Then } f(x) \text{ is}$$

(A) $\frac{1}{3x} + \frac{2x^2}{3}$ (B) $\frac{-1}{3x} + \frac{4x^2}{3}$ (C) $\frac{-1}{x} + \frac{2}{x^2}$ (D) $\frac{1}{x}$

[JEE Advanced 2007; Paper 1:Straight Objective Type]

33. If f is a real-valued differentiable function satisfying $|f(x) - f(y)| \leq (x-y)^2, x, y \in R$ and $f(0) = 0$, then $f(1)$ equals

(A) -1 (B) 0 (C) 2 (D) 1 **[AIEEE 2005]**

34. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h \to 0} \frac{1}{h} f(1+h) = 5$, then $f'(1)$ equals

(A) 3 (B) 4 (C) 5 (D) 6 **[AIEEE 2005]**

35. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then $f(x)$ is

(A) discontinuous everywhere (B)continuous and differentiable for all x.

(C)continuous for all x but not differentiable at $x = 0$. (D) neither differentiable nor continuous at $x = 0$.

[AIEEE 2003]

Differentiation

LEVEL 1 (*NCERT Based*)

36. If $x = a\cos\theta$, $y = a\sin\theta$, then $\frac{d^2y}{dx^2} =$ ____ . $(\text{where } a \neq 0, \theta \neq k\pi, k \in z)$

(A) $-\frac{1}{a^2}\cos ec^2\theta \sec\theta$ (B) $\frac{1}{a}\cot^3\theta$ (C) $\cos ec^2\theta$ (D) $-\frac{1}{a}\cos ec^3\theta$

[GUJCET 2023]

37. If $y = \sqrt{\sin^{-1}x + y}$, then $\frac{dy}{dx} =$_____. $(\text{where } x \in (0,1))$

(A) $\frac{1}{(2y-1)\sqrt{1-x^2}}$ (B) $\frac{1}{(1-2y)\sqrt{1-x^2}}$ (C) $\frac{1}{(2y-1)\sqrt{x^2-1}}$ (D) $\frac{1}{(2y+1)\sqrt{1-x^2}}$

[GUJCET 2023]

38. If $\left\{\frac{d}{dx}\left(x^x + x^{x+1} + x^{x+2}\right)\right\}_{x=e} =$ _____

(A) $e^e\left(1+e^2+2e\right)$ (B) $e^e\left(3e^2+2e+2\right)$ (C) $e^e\left(2e^2+4e+3\right)$ (D) $e^e\left(1+4e+2e^2\right)$

[GUJCET 2023]

39. If $x = \sqrt{10^{\sin^{-1}t}}$, $y = \sqrt{10^{\cos^{-1}t}}$ then $\frac{dy}{dx} =$_____

(A) $-\frac{x}{y}$ (B) $\frac{y}{x}$ (C) 0 (D) $-\frac{y}{x}$ **[GUJCET 2022]**

40. If $y = 100\,e^{2x} + 200\,e^{-2x}$ and $\frac{d^2y}{dx^2} = ay$ then $a =$ ____

(A) 4 (B) -4 (C) 2 (D) 0 **[GUJCET 2022]**

41.If $x = \sqrt{2^{\cos ec^{-1}t}}$ and $y = \sqrt{2^{\sec^{-1}t}}$ $(|t| \geq 1)$ then $\frac{dy}{dx}$ is equal to:

(A) $\frac{y}{x}$ (B) $\frac{x}{y}$ (C) $-\frac{y}{x}$ (D) $-\frac{x}{y}$ **[JEE Main Online 2018]**

42. If $\sin y = x\sin(a+y)$, then $\frac{dy}{dx}$ is

(A) $\frac{\sin a}{\sin^2(a+y)}$ (B) $\frac{\sin^2(a+y)}{\sin a}$ (C) $\sin a \sin^2(a+y)$ (D) $\frac{\sin^2(a-y)}{\sin a}$ **[AIEEE 2002]**

43. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is

(A) $\frac{1+x}{1+\log x}$ (B) $\frac{1-\log x}{1+\log x}$ (C) not defined (D) $\frac{\log x}{(1+\log x)^2}$ **[AIEEE 2002]**

LEVEL 2 (*NCERT* plus)

44. If $y(x)=x^x,\ x>0$, then $y''(2)-2y'(2)$ is equal to:

(A) $8\log_e 2-2$ (B) $4\log_e 2+2$ (C) $4(\log_e 2)^2+2$ (D) $4(\log_e 2)^2-2$

[JEE Main 2023, 1 February (E)]

45. If $2x^y+3y^x=20$, then $\frac{dy}{dx}$ at $(2,2)$ is equal to:

(A) $-\left(\frac{3+\log_e 4}{2+\log_e 8}\right)$ (B) $-\left(\frac{3+\log_e 16}{4+\log_e 8}\right)$ (C) $-\left(\frac{3+\log_e 8}{2+\log_e 4}\right)$ (D) $-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$

[JEE Main 2023, 6 April (M)]

46.If $x^2+y^2+\sin y=4$, then the value of $\frac{d^2y}{dx^2}$ at the point $(-2,0)$ is:.

(A) -34 (B) -32 (C) 4 (D)-2 **[JEE Main 2018]**

47.If $f(x)=\sin^{-1}\left(\frac{2\times 3^x}{1+9^x}\right)$, then $f'\left(-\frac{1}{2}\right)$ equals :

(A) $-\sqrt{3}\log_e\sqrt{3}$ (B) $\sqrt{3}\log_e\sqrt{3}$ (C) $-\sqrt{3}\log_e 3$ (D) $\sqrt{3}\log_e 3$

[JEE Main 2018]

48.If for $x\in\left(0,\frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}.g(x)$, then $g(x)$ equals:

(A) $\frac{3x\sqrt{x}}{1-9x^3}$ (B) $\frac{3x}{1-9x^3}$ (C) $\frac{3x}{1+9x^3}$ (D) $\frac{9}{1+9x^3}$

[JEE Main 2017]

49. If g is the inverse of a function f and $f'(x)=\frac{1}{1+x^5}$, then $g'(x)$ is

(A) $1+x^5$ (B) $5x^4$ (C) $\frac{1}{1+\{g(x)\}^5}$ (D) $1+\{g(x)\}^5$

[JEE Main 2014]

50. If $y=e^{nx}$, then $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right)$ is equal to:

(A) ne^{nx} (B) ne^{-nx} (C) 1 (D) $-ne^{-nx}$

[JEE Main 2014]

51. If $f(x) = x^2 - x + 5, x > \frac{1}{2}$ and $g(x)$ is its inverse function, then $g'(7)$ equals :

(A) -1/3 (B) 1/13 (C)1/3 (D)-1/13

[JEE Main Online 2014]

52. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to

(A) $\frac{1}{2}$ (B) 1 (C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$ **[JEE Main 2013]**

53. Let $f(x) = \frac{x^2 - x}{x^2 + 2x}, x \neq 0, -2$. Then $\frac{d}{dx}\left[f^{-1}(x)\right]$ (whenever it is defined) is equal to :

(A) $\frac{-1}{(1-x)^2}$ (B) $\frac{3}{(1-x)^2}$ (C) $\frac{1}{(1-x)^2}$ (D) $\frac{-3}{(1-x)^2}$

[JEE Main Online 2013]

54. If $f(x) = \sin(\sin x)$ and $f''(x) + \tan x f'(x) + g(x) = 0$, then $g(x)$ is:

(A) $\cos^2 x \cos(\sin x)$ (B) $\sin^2 x \cos(\cos x)$ (C) $\sin^2 x \sin(\cos x)$ (D) $\cos^2 x \sin(\sin x)$

[JEE Main Online 2013]

55. Let $f(\theta) = \sin\left[\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right]$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is

[JEE Advanced 2011; Paper 1: Integer Answer Type]

56. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals:

(A)1 (B) $\log 2$ (C) $-\log 2$ (D) -1 **[AIEEE 2009]**

57. If $x^m . y^n = (x + y)^{m+n}$, then $\frac{dy}{dx}$ is

(A) $\frac{x+y}{xy}$ (B) xy (C) $\frac{x}{y}$ (D) $\frac{y}{x}$ **[AIEEE 2006]**

58. If $x = e^{y + e^{y + \ldots \infty}}$, $x > 0$ then $\frac{dy}{dx}$ is

(A) $\frac{x}{1+x}$ (B) $\frac{1}{x}$ (C) $\frac{1-x}{x}$ (D) $\frac{1+x}{x}$ **[AIEEE 2004]**

59. If $y = \left(x + \sqrt{1 + x^2}\right)^n$, then $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is

(A) $n^2 y$ (B) $-n^2 y$ (C) $-y$ (D) $2x^2 y$ **[AIEEE 2002]**

LEVEL 3 (*Non NCERT Based*)

60.Let $S=\left\{t\in R: f(x)=|x-\pi|.\left(e^{|x|}-1\right).\sin|x| \text{ is not } differentiable \text{ at t}\right\}$. Then the set S is equal to:

(A) φ (an empty set) (B) $\{0\}$ (C) $\{\pi\}$ (D) $\{0,\pi\}$ **[JEE Main 2018]**

61.Let $S=\left\{(\lambda,\mu)\in R\times R: f(t)=\left(|\lambda|e^{|t|}-\mu\right).\sin(2|t|), t\in R, \text{is a } differentiable \text{ function}\right\}$. Then S is a subset of :

(A) $R\times[0,\infty)$ (B) $[0,\infty)\times R$ (C) $R\times(-\infty,0)$ (D) $(-\infty,0)\times R$

[JEE Main Online 2018]

62.Let $f(x)$ be a polynomial of degree 4 having extreme values at $x=1$ and $x=2$. If $\lim\limits_{x\to 0}\left(\frac{f(x)}{x^2}+1\right)=3$ then $f(-1)$ is equal to:

(A) $\frac{9}{2}$ (B) $\frac{5}{2}$ (C) $\frac{3}{2}$ (D) $\frac{1}{2}$

[JEE Main Online 2018]

63.If $y=\left[x+\sqrt{x^2-1}\right]^{15}+\left[x-\sqrt{x^2-1}\right]^{15}$, then $\left(x^2-1\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}$ is equal to :

(A) $125y$ (B) $225y^2$ (C) $225y$ (D) $224y^2$

[JEE Main Online 2017]

64.Let f be a polynomial function such that $f(3x)=f'(x).f''(x)$, for all $x\in R$. Then:

(A) $f(2)-f'(2)+f''(2)=10$ (B) $f''(2)-f(2)=4$

(C) $f''(2)-f'(2)=0$ (D) $f(2)+f'(2)=28$ **[JEE Main Online 2017]**

65. Let f and g be two differentiable functions on R such that $f'(x)>0$ and $g'(x)<0$,for all $x\in R$. Then for all x:

(A) $f(g(x))>f(g(x-1))$ (B) $f(g(x))>f(g(x+1))$

(C) $g(f(x))>g(f(x-1))$ (D) $g(f(x))>g(f(x+1))$ **[JEE Main Online 2014]**

66. Let $f(1)=-2$ and $f'(x)\geq 4.2$ for $1\leq x\leq 6$ .Then possible value of $f(6)$ lies in the interval:

(A)[15,19) (B) $(-\infty,12)$ (C)[12,15) (D)$[19,\infty)$ **[JEE Main Online 2013]**

67. $\frac{d^2x}{dy^2}$ equals :

(A) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (B) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ (C) $\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-2}$ (D) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

[AIEEE 2011]

68. Let $f:(-1,1)\to R$ be a differentiable function with $f(0)=-1$ and $f'(0)=1$. Let $g(x)=\left[f\left(2f(x)+2\right)\right]^2$. Then $g'(0)=$

(A) -4 (B) 0 (C) -2 (D) 4 **[AIEEE 2010]**

69. $\dfrac{d^2x}{dy^2}$ equals

(A) $\left(\dfrac{d^2y}{dx^2}\right)^{-1}$ (B) $-\left(\dfrac{d^2y}{dx^2}\right)^{-1}\left(\dfrac{dy}{dx}\right)^{-3}$ (C) $\left(\dfrac{d^2y}{dx^2}\right)^{-1}\left(\dfrac{dy}{dx}\right)^{-2}$ (D) $-\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-3}$

[JEE 2007 Advanced; Paper 2: Straight Objective Type]

70. Let $f(x)$ be a polynomial function of second degree. If $f(1)=f(-1)$ and a, b, c are in AP, then $f'(a)$, $f'(b)$ and $f'(c)$ are in

(A) AP (B) GP (C)HP (D) Arithmetico-Geometric Progression.

[AIEEE 2003]

71. If $f(x)=x^n$, then the value of $f(1)-\dfrac{f'(1)}{1!}+\dfrac{f''(1)}{2!}-\dfrac{f'''(1)}{3!}+.....+\dfrac{(-1)^n f^n(1)}{n!}$ is

(A) $2^{n\,n}$ (B) 2^{n-1} (C) 0 (D) 1 **[AIEEE 2003]**