

ASSIGNMENT**CHAPTER 5: Continuity & Differentiability**

CLASS: 12 NCERT

SUBJECT: MATHEMATICS

Assignment 5.1**Continuity and Differentiability**

* Important points to remember:

Continuity at point c :

Definition : Suppose f is a real function on a subset of the real numbers and let c be a point in the domain of f . Then f is continuous at c if $\lim_{x \rightarrow c} f(x) = f(c)$.

Differentiability:* Suppose f is a real valued function and c is a point in its domain. The derivative of f

at c is defined by $f'(c) = \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}$ provided this limit exists.

* Suppose f is a real valued function, the function defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

wherever the limit exists is defined to be the derivative of f at x .

NOTE: Every differentiable function is continuous, but the converse is not true.

.....**Questions from CBSE Question paper**.....

[**1** mark Questions]

1. Determine the value of 'k' for which the following function is continuous at $x = 3$:

$$f(x) = \begin{cases} \frac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$

[Ans: $k = 12$]

[2017]

[**4** marks Questions]

2. If $f(x) = \begin{cases} \frac{\sin(a+1)x + 2 \sin x}{x}, & x < 0 \\ 2, & x = 0 \\ \frac{\sqrt{1+bx} - 1}{x}, & x > 0 \end{cases}$ is continuous at $x = 0$, then find the values of

a and b .

[Ans: $a = -1, b = 4$]

[2016]

3. Discuss the continuity and differentiability of the function $f(x) = |x| + |x-1|$ in the interval $(-1, 2)$.

[Ans: Given function is continuous but not differentiable at 0 and 1] [All India 2015]

4. Find the value of the constant k so that the function f , defined below, is continuous at $x = 0$, where

$$f(x) = \begin{cases} \left(\frac{1 - \cos 4x}{8x^2} \right), & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$$

[Ans: 1]

[All India 2014C]

5. If $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}, & \text{when } x < 0 \\ a, & \text{when } x = 0 \text{ and } f \text{ is continuous at } x = 0, \text{ then find } a. \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}, & \text{when } x > 0 \end{cases}$

[Ans: 8]

[Delhi 2013C]

6. Show that the function $f(x) = |x - 3|$, $x \in \mathbb{R}$, is continuous but not differentiable at $x = 3$.
[Delhi 2013]

7. Find the value of k , for which $f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-1}, & \text{if } 0 \leq x < 1 \end{cases}$ is continuous

at $x = 0$.

[Ans: -1]

[All India 2013]

8. Find the value of k , so that the function f defined by

$$f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases} \text{ is continuous at } x = \frac{\pi}{2}.$$

[Ans: 6]

[Delhi 2012C, Foreign 2011]

9. Find the value of a for which the function f is defined as

$$f(x) = \begin{cases} a \sin \frac{\pi}{2}(x+1), & x \leq 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases} \text{ is continuous at } x = 0.$$

[Ans: $\frac{1}{2}$]

[Delhi 2011]

10. If the function $f(x)$ given by

$$f(x) = \begin{cases} 3ax + b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases} \text{ is continuous at } x = 1, \text{ then find the values of } a \text{ and } b.$$

[Ans: $a = 3; b = 2$]

[Delhi 2011; All India 2010]

11. Find the values of a and b such that the following function $f(x)$ is a continuous function.

$$f(x) = \begin{cases} 5, & x \leq 2 \\ ax + b, & 2 < x < 10 \\ 21, & x \geq 10 \end{cases}$$

[Ans: $a = 2; b = 1$]

[Delhi 2011]

12. Find the relationship between a and b , so that the function f defined by

$$f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases} \text{ is continuous at } x = 3.$$

[Ans: $3a - 3b = 2$]

[All India 2011]

13. Find the value of k , so that the function f defined by

$$f(x) = \begin{cases} kx + 1, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases} \text{ is continuous at } x = \pi$$

[Ans: $-\frac{2}{\pi}$]

[Foreign 2011]

14. For what values of λ , is the function

$$f(x) = \begin{cases} \lambda(x^2 - 2x), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x > 0 \end{cases} \text{ is continuous at } x = 0?$$

[Ans: No value of λ exists for given function to be continuous]

[Foreign 2011]

15. Discuss the continuity of the function $f(x)$ at $x = 1/2$, when $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 1/2 + x, & 0 \leq x < 1/2 \\ 1, & x = 1/2 \\ 3/2 + x, & 1/2 < x \leq 1 \end{cases}$$

[Ans: $f(x)$ is discontinuous at $x = \frac{1}{2}$]

[Delhi 2011C]

16. Find the value of a , if the function $f(x)$ defined by

$$f(x) = \begin{cases} 2x - 1, & x < 2 \\ a, & x = 2 \\ x + 1, & x > 2 \end{cases} \text{ is continuous at } x = 2.$$

[Ans: 3]

[All India 2011C]

17. Find the values of a and b such that the function defined as follows is continuous

$$f(x) = \begin{cases} x + 2, & x \leq 2 \\ ax + b, & 2 < x < 5 \\ 3x - 2, & x \geq 5 \end{cases}$$

[Ans: $a = 3, b = -2$]

[Delhi 2010, 2010C]

18. For what value of k is the function defined by

$$f(x) = \begin{cases} k(x^2 + 2), & \text{if } x \leq 0 \\ 3x + 1, & \text{if } x > 0 \end{cases} \text{ continuous at } x = 0?$$

Also, write whether the function is continuous at $x = 1$.

[Ans: $k = \frac{1}{2}$, $f(x)$ is continuous at $x = 1$] [Delhi 2010, 2010C]

19. Find all points of discontinuity of f , where f is defined as follows:

$$f(x) = \begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases}$$

[Ans: $f(x)$ is continuous at $x = -3$ and discontinuous at $x = 3$] [Delhi 2010]

20. Show that the function defined as follows, is continuous at $x = 1, x = 2$ but not

differentiable at $x = 2$. $f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 - x, & 1 < x \leq 2 \\ 5x - 4, & x > 2 \end{cases}$ [Delhi 2010]

21. Show that the function $f(x)$ defined by

$$f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x > 0 \\ 2, & x = 0 \\ \frac{4(1 - \sqrt{1-x})}{x}, & x < 0 \end{cases} \text{ is continuous at } x = 0.$$

[Ans: $f(x)$ is continuous at $x = 0$] [All India 2009C]

22. If the function defined by $f(x) = \begin{cases} 2x - 1, & x < 2 \\ a, & x = 2 \\ x + 1, & x > 2 \end{cases}$ is continuous at $x = 2$, then find the

value of a . Also discuss the continuity of $f(x)$ at $x = 3$.

[Ans: $a = 3$; $f(x)$ is continuous at $x = 3$] [Delhi 2009C]

23. For what value of k is the following function continuous at $x = 2$?

$$f(x) = \begin{cases} 2x+1, & x < 2 \\ k, & x = 2 \\ 3x-1, & x > 2 \end{cases}$$

[Ans: $k = 5$]

[Delhi 2008]

24. If $f(x)$ defined by the following, is continuous at $x = 0$, then find the value of a, b and c .

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & \text{if } x < 0 \\ c, & \text{if } x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}}, & \text{if } x > 0 \end{cases}$$

[Ans: $a = -\frac{3}{2}, b \in R, c = \frac{1}{2}$]

[All India 2008]

25. Show that $f(x) = \begin{cases} 5x-4, & \text{when } 0 < x \leq 1 \\ 4x^3 - 3x, & \text{when } 1 < x < 2 \end{cases}$ is continuous at $x = 1$.

[Delhi 2008C]

26. If the following function $f(x)$ is continuous at $x = 0$, then find the value of k

$$f(x) = \begin{cases} \frac{1 - \cos 2x}{2x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

[Ans: $k = 1$]

[All India 2008C]

Assignment 5.2

Differentiation

1-21: Important formulae:

$$1. \frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$2. \frac{d(u.v)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$3. \frac{d(u-v)}{dx} = \frac{du}{dx} - \frac{dv}{dx}$$

$$4. \frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$5. \frac{d(\text{const } t)}{dx} = 0$$

$$6. \frac{d(u^n)}{dx} = nu^{n-1} \cdot \frac{du}{dx}$$

$$7. \frac{d(a^u)}{dx} = a^u \cdot \log_e a \cdot \frac{du}{dx}, a > 0$$

$$8. \frac{d(e^u)}{dx} = e^u \cdot \frac{du}{dx}$$

$$9. \frac{d(\sin u)}{dx} = \cos u \cdot \frac{du}{dx}$$

$$10. \frac{d(\cos u)}{dx} = -\sin u \cdot \frac{du}{dx}$$

$$11. \frac{d(\tan u)}{dx} = \sec^2 u \cdot \frac{du}{dx}$$

$$12. \frac{d(\sec u)}{dx} = \sec u \cdot \tan u \cdot \frac{du}{dx}$$

$$13. \frac{d(\cot u)}{dx} = -\operatorname{cosec}^2 u \cdot \frac{du}{dx}$$

$$14. \frac{d(\operatorname{cosec} u)}{dx} = -\operatorname{cosec} u \cdot \cot u \cdot \frac{du}{dx}$$

$$15. \frac{d(\sin^{-1} u)}{dx} = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

$$16. \frac{d(\cos^{-1} u)}{dx} = \frac{-1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

$$17. \frac{d(\tan^{-1} u)}{dx} = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

$$18. \frac{d(\cot^{-1} u)}{dx} = \frac{-1}{1+u^2} \cdot \frac{du}{dx}$$

$$19. \frac{d(\sec^{-1} u)}{dx} = \frac{1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx}$$

$$20. \frac{d(\operatorname{cosec}^{-1} u)}{dx} = \frac{-1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx}$$

$$21. \frac{d(\log u)}{dx} = \frac{1}{u} \cdot \frac{du}{dx}$$

22-29: Logarithm and its properties:

22. $\log_a b = x \Leftrightarrow a^x = b$. (a is known as the base of the logarithm)

23. $\log_a 1 = 0$

24. $\log_a a = 1$

25. $\log_a (m \times n) = \log_a m + \log_a n$

26. $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

27. $\log_a m^n = n \log_a m$

28. $a^{\log_a x} = x$

29. $\log_a b = \frac{\log_c b}{\log_c a}$ (Base change formula).

30-33: Following formulae needs special attention:

BASE	POWER	DIFFERENTIATION
30. Variable	Constant	$\frac{d(u^n)}{dx} = nu^{n-1} \frac{du}{dx}$
31. Variable	Variable	$\frac{dx^x}{dx} = x^x(1 + \log x)$
32. Constant	Constant	$\frac{d(\text{constant})}{dx} = 0$
33. Constant	Variable	$\frac{d(a^u)}{dx} = a^u \log_e a \frac{du}{dx}, a > 0$

.....**Questions from CBSE Question paper**

1. If $y = (x + \sqrt{x^2 - 1})^2$, then show that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$.

[CBSE 2023, Set 1; 2 marks]

2. If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is:

(A) $-\sec^2\left(\frac{\pi}{4} - x\right)$

(B) $\sec^2\left(\frac{\pi}{4} - x\right)$

(C) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$

(D) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$

[CBSE 2023, Set 3; 1 mark]

3. If $y = \sin^2(x^3)$, then $\frac{dy}{dx}$ is equal to :

(A) $2 \sin x^3 \cos x^3$

(B) $3x^3 \sin x^3 \cos x^3$

(C) $6x^2 \sin x^3 \cos x^3$

(D) $2x^2 \sin^2(x^3)$

[CBSE 2023, Set 4; 1 mark]

⇒ **Derivatives of Implicit Functions**

4. If $(x^2 + y^2)^2 = xy$, then find $\frac{dy}{dx}$

[CBSE 2023, Set 4; 2 marks]

5. If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to

(A) $-\frac{y}{x}$

(B) $\frac{y}{x}$

(C) $\sec^2\left(\frac{y}{x}\right)$

(D) $-\sec^2\left(\frac{y}{x}\right)$

[CBSE 2023, Set 2; 1 mark]

⇒ **Derivatives of Inverse Functions**

6. Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.

[CBSE 2023, Set 5; 3 marks]

⇒ **Exponential and Logarithmic Functions**

7. If $y = x^{\frac{1}{x}}$, then find $\frac{dy}{dx}$ at $x = 1$.

[CBSE 2023, Set 3; 2 marks]

8. The derivative of x^{2x} w.r.t. x is

(A) x^{2x-1}

(B) $2x^{2x} \log x$

(C) $2x^{2x}(1 + \log x)$

(D) $2x^{2x}(1 - \log x)$

[CBSE 2023, Set 5; 1 mark]

9. If $(\cos x)^y = (\cos y)^x$, then $\frac{dy}{dx}$ is equal to:

- (A) $\frac{y \tan x + \log(\cos y)}{x \tan y - \log(\cos x)}$ (B) $\frac{x \tan y + \log(\cos x)}{y \tan x + \log(\cos y)}$
 (C) $\frac{y \tan x - \log(\cos y)}{x \tan y - \log(\cos x)}$ (D) $\frac{y \tan x + \log(\cos y)}{x \tan y + \log(\cos x)}$ [CBSE 2023C; 1 mark]

10. If $y(x) = x^x$, $x > 0$, then $y''(2) - 2y'(2)$ is equal to:

- (A) $8 \log_e 2 - 2$ (B) $4 \log_e 2 + 2$ (C) $4(\log_e 2)^2 + 2$ (D) $4(\log_e 2)^2 - 2$
 [JEE Main 2023, 1 February (E)]

11. If $2x^y + 3y^x = 20$, then $\frac{dy}{dx}$ at $(2,2)$ is equal to:

- (A) $-\left(\frac{3 + \log_e 4}{2 + \log_e 8}\right)$ (B) $-\left(\frac{3 + \log_e 16}{4 + \log_e 8}\right)$ (C) $-\left(\frac{3 + \log_e 8}{2 + \log_e 4}\right)$ (D) $-\left(\frac{2 + \log_e 8}{3 + \log_e 4}\right)$
 [JEE Main 2023, 6 April (M)]

⇒ **Derivatives of Functions in Parametric Forms**

12. If $x = a \sin 2t$, $y = a(\cos 2t + \log \tan t)$, then find $\frac{dy}{dx}$. [CBSE 2023, Set 3; 2 marks]

13. If $x = a \cos \theta + b \sin \theta$, $y = a \sin \theta - b \cos \theta$, then which one of the following is true?

- (A) $y^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 0$ (B) $y^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$
 (C) $y^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = 0$ (D) $y^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - y = 0$ [CBSE 2023C; 1 mark]

⇒ **Second Order Derivative**

14. If $y = \sqrt{ax + b}$, prove that $y \left(\frac{d^2 y}{dx^2} \right) + \left(\frac{dy}{dx} \right)^2 = 0$. [CBSE 2023, Set 2; 2 marks]

15. If $x = A \cos 4t + B \sin 4t$, then $\frac{d^2 x}{dt^2}$ is equal to

- (A) x (B) $-x$ (C) $16x$ (D) $-16x$
 [CBSE 2023, Set 5; 1 mark]

16. If $y = \tan x + \sec x$, then prove that $\frac{d^2 y}{dx^2} = \frac{\cos x}{(1 - \sin x)^2}$. [CBSE 2023, Set 5; 3 marks]

[2 marks Questions]

1. Differentiate $\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ with respect to x .

[Ans: $-\frac{1}{2}$]

[2018]

[4 marks Questions]

2. If $(\sin x)^y = x + y$, find $\frac{dy}{dx}$.

[2019]

[Ans: $\frac{1-y(x+y)\cot x}{(x+y)\log \sin x - 1}$]

3. If $y = (\sec^{-1} x)^2$, $x > 0$, show that $x^2(x^2 - 1)\frac{d^2y}{dx^2} + (2x^3 - x)\frac{dy}{dx} - 2 = 0$

[2019]

4. Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left[\frac{2^{x+1}}{1+4^x}\right]$

[2019]

[Ans: $\frac{2^{x+1} \cdot \log 2}{1+4^x}$]

5. If $(x^2 + y^2)^2 = xy$, find $\frac{dy}{dx}$.

[Ans: $\frac{4x(x^2 + y^2) - y}{x - 4y(x^2 + y^2)}$]

[2018]

6. If $x = a(2\theta - \sin 2\theta)$ and $y = a(1 - \cos 2\theta)$, find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$.

[Ans: $\frac{1}{\sqrt{3}}$]

[2018]

7. If $y = \sin(\sin x)$, prove that $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$.

[2018]

8. If $x^y + y^x = a^b$, then find $\frac{dy}{dx}$.

[Ans: $\frac{-[y^x \log y + y \cdot x^{y-1}]}{x \cdot y^{x-1} + x^y \cdot \log x}$]

[2017]

9. If $e^y(x+1) = 1$, then prove $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$. [2019, 2017]

10. If $x \cos(a+y) = \cos y$ then prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

Hence show that $\sin a \frac{d^2y}{dx^2} + \sin 2(a+y) \frac{dy}{dx} = 0$. [2016]

11. Find $\frac{dy}{dx}$ if $y = \sin^{-1} \left[\frac{6x - 4\sqrt{1-4x^2}}{5} \right]$.

[Ans: $\frac{2}{\sqrt{1-4x^2}}$] [2016]

12. If $x = a(\cos 2t + 2t \sin 2t)$ and $y = a(\sin 2t - 2t \cos 2t)$, then find $\frac{d^2y}{dx^2}$.

[Ans: $\frac{\sec^3 2t}{2at}$] [All India 2015]

13. If $(ax+b)e^{y/x} = x$, then show that $x^3 \left(\frac{d^2y}{dx^2} \right) = \left(x \frac{dy}{dx} - y \right)^2$. [All India 2015]

14. Differentiate $\tan^{-1} \left(\frac{\sqrt{1-x^2}}{x} \right)$ w.r.t. $\cos^{-1}(2x\sqrt{1-x^2})$, when $x \neq 0$.

[Ans: $-\frac{1}{2}$] [All India 2015]

15. If $y = x^x$, then prove that $\frac{d^2y}{dx^2} - \frac{1}{y} \left(\frac{dy}{dx} \right)^2 - \frac{y}{x} = 0$. [Delhi 2014]

16. Differentiate $\tan^{-1} \left(\frac{x}{\sqrt{1-x^2}} \right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$, when $x \neq 0$.

[Ans: $\frac{1}{2}$] [Delhi 2014]

17. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, when $x \neq 0$.

[Ans: $\frac{1}{4}$]

[Delhi 2014]

18. Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$, if $x = ae^\theta(\sin\theta - \cos\theta)$ and $y = ae^\theta(\sin\theta + \cos\theta)$.

[Ans: 1]

[All India 2014]

19. If $y = Pe^{ax} + Qe^{bx}$, then show that $\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = 0$. [All India 2014]

20. If $x = a \sin 2t(1 + \cos 2t)$ and $y = b \cos 2t(1 - \cos 2t)$, then show that at $t = \frac{\pi}{4}$,

$$\left(\frac{dy}{dx}\right) = \frac{b}{a}.$$

[All India 2014]

21. If $x = \cos t(3 - 2\cos^2 t)$ and $y = \sin t(3 - 2\sin^2 t)$, then find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

[Ans: 0]

[All India 2014]

22. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, then prove that $\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$.

[Delhi 2013C]

23. If $y = (\sin x)^x + \sin^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$.

$$\text{[Ans: } (\sin x)^x [x \cot x + \log \sin x] + \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} \text{]}$$

[Delhi 2013C, 2009; All India 2009C]

24. If $y = x \log\left(\frac{x}{a+bx}\right)$, then prove that $x^3 \frac{d^2y}{dx^2} = \left(x \frac{dy}{dx} - y\right)^2$.

[Delhi 2013C]

25. Differentiate the following function with respect to x . $(\log x)^x + x^{\log x}$.

$$\text{[Ans: } (\log x)^x \left\{ \frac{1}{\log x} + \log(\log x) \right\} + 2 \left(\frac{\log x}{x} \right) x^{\log x} \text{]}$$

[Delhi 2013]

26. If $y = \log \left[x + \sqrt{x^2 + a^2} \right]$, then show that $(x^2 + a^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 0$ [Delhi 2013]

27. If $x = a \sin t$ and $y = a \left(\cos t + \log \tan \frac{t}{2} \right)$, then find $\frac{d^2y}{dx^2}$.

[Ans: $-\frac{\operatorname{cosec}^2 t}{a \cos t}$]

[Delhi 2013]

28. Differentiate the following with respect to x :

$$\sin^{-1} \left[\frac{2^{x+1} \cdot 3^x}{1 + (36)^x} \right]$$

[Ans: $\left[\frac{2^{x+1} \cdot 3^x}{1 + (36)^x} \right] \cdot \log 6$]

[All India 2013]

29. If $x = a \cos^3 \theta$ and $y = a \sin^3 \theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$.

[Ans: $\frac{32}{27a}$]

[All India 2013]

30. If $x \sin(a + y) + \sin a \cos(a + y) = 0$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$.

[All India 2013]

31. If $x^y = e^{x-y}$, then prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$.

[All India 2013]

32. If $y^x = e^{y-x}$, then prove that $\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}$.

[All India 2013]

33. If $(\cos x)^y = (\cos y)^x$, then find $\frac{dy}{dx}$.

[Ans: $\frac{\log(\cos y) + y \tan x}{x \tan y + \log(\cos x)}$]

[Delhi 2012]

34. If $\sin y = x \sin(a + y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$. [2019, Delhi 2012]

35. If $x = \sqrt{a^{\sin^{-1} t}}$ and $y = \sqrt{a^{\cos^{-1} t}}$, then show that $\frac{dy}{dx} = \frac{-y}{x}$. [All India 2012]

36. Differentiate $\tan^{-1} \left[\frac{\sqrt{1+x^2} - 1}{x} \right]$ w.r.t. x .

[Ans: $\frac{1}{2(1+x^2)}$] [All India 2012]

37. If $y = (\tan^{-1} x)^2$, then show that $(x^2 + 1)^2 \frac{d^2 y}{dx^2} + 2x(x^2 + 1) \frac{dy}{dx} = 2$. [Delhi 2012]

38. If $y = x^{\sin x - \cos x} + \frac{x^2 - 1}{x^2 + 1}$, then find $\frac{dy}{dx}$.

[Ans: $x^{\sin x - \cos x} \left[\frac{\sin x - \cos x}{x} + \log x (\cos x + \sin x) \right] + \frac{4x}{(x^2 + 1)^2}$] [Delhi 2012 C]

39. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, then find $\frac{d^2 y}{dx^2}$ and $\frac{d^2 y}{dt^2}$.

[Ans: $\frac{\sec^3 t}{at}$; $a(\sin t + t \cos t)$] [Delhi 2012C]

40. Find $\frac{dy}{dx}$, when $y = x^{\cot x} + \frac{2x^2 - 3}{x^2 + x + 2}$.

[Ans: $x^{\cot x} \left(\frac{\cot x}{x} - \operatorname{cosec}^2 x \cdot \log x \right) + \frac{2x^2 + 14x + 3}{(x^2 + x + 2)^2}$] [All India 2012C]

.....**REVISION SESSION**[4 marks each].....

1. Discuss the continuity and differentiability of the function $f(x) = |x| + |x - 1|$ in the interval $(-1, 2)$.
[Ans: Given function is continuous but not differentiable at 0 and 1]
2. If $x = a(\cos 2t + 2t \sin 2t)$ and $y = a(\sin 2t - 2t \cos 2t)$, then find $\frac{d^2y}{dx^2}$. [Ans: $\frac{\sec^3 2t}{2at}$]
3. If $(ax + b)e^{y/x} = x$, then show that $x^3 \left(\frac{d^2y}{dx^2} \right) = \left(x \frac{dy}{dx} - y \right)^2$.
4. Show that the function $f(x) = |x - 1| + |x + 1|$, for all $x \in R$, is not differentiable at the points $x = -1$ and $x = 1$.
5. If $y = e^{m \sin^{-1} x}$, then show that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - my^2 = 0$.
6. If $f(x) = \sqrt{x^2 + 1}$; $g(x) = \frac{x+1}{x^2 + 1}$ and $h(x) = 2x - 3$, then find $f'[h'\{g'(x)\}]$.
[Ans: $\frac{2}{\sqrt{5}}$]
7. Let $f(x) = x - |x - x^2|$, $x \in [-1, 1]$. Find the point of discontinuity, (if any), of this function on $[-1, 1]$.
Ans: No point of discontinuity.
8. If $y = \log \left(\frac{x}{a + bx} \right)^x$, prove that $x^3 \frac{d^2y}{dx^2} = \left(x \frac{dy}{dx} - y \right)^2$.
9. Find the derivative of $\sec^{-1} \left(\frac{1}{2x^2 - 1} \right)$ w.r.t. $\sqrt{1 - x^2}$ at $x = \frac{1}{2}$.
Ans: 4
10. If $y = (3 \cot^{-1} x)^2$, show that $(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1) \frac{dy}{dx} = 18$.
11. Show that the function $f(x) = |x - 3|$, $x \in R$, is continuous but not differentiable at $x = 3$.

12. If $y = \left(x + \frac{1}{x}\right)^x + x^{\left(1 + \frac{1}{x}\right)}$, find $\frac{dy}{dx}$.

Ans: $\left(x + \frac{1}{x}\right)^x \left[\log\left(x + \frac{1}{x}\right) + \frac{x^2 - 1}{x^2 + 1} \right] + x^{\left(1 + \frac{1}{x}\right)} \left[\frac{x^2 + 1 - \log x}{x^2} \right]$

13. If $y = x^3 \log\left(\frac{1}{x}\right)$, then prove that $x \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 3x^2 = 0$.

14. If $y = \frac{x \cos^{-1} x}{\sqrt{1-x^2}} - \log \sqrt{1-x^2}$, then prove that $\frac{dy}{dx} = \frac{\cos^{-1} x}{(1-x^2)^{3/2}}$.

15. Find the derivative of $(\sin x)^x + \sin^{-1} \sqrt{x}$ w. r. t. x .

[Ans: $(\sin x)^x [x \cot x + \log \sin x] + \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$]

16. If $x = a \sec^3 \theta$, $y = \tan^3 \theta$, then find $\frac{d^2 y}{dx^2}$ at $\theta = \frac{\pi}{4}$. **[Ans:** $\frac{1}{12a}$]

17. For what value of λ the function defined by $f(x) = \begin{cases} \lambda(x^2 + 2), & \text{if } x \leq 0 \\ 4x + 6, & \text{if } x > 0 \end{cases}$ is

continuous at $x = 0$? Hence check the differentiability of $f(x)$ at $x = 0$.

[Ans: $\lambda = 3$, $f(x)$ is not differentiable at $x = 0$.]

18. If $x = ae^t (\sin t + \cos t)$ and $y = ae^t (\sin t - \cos t)$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

19. If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2 y}{dx^2} - (m+n) \frac{dy}{dx} + mny = 0$.

20. If $y = \tan^{-1} \left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right)$, $x^2 \leq 1$, then find $\frac{dy}{dx}$. **[Ans:** $-\frac{x}{\sqrt{1-x^4}}$]

21. If $x = a \cos \theta + b \sin \theta$, $y = a \sin \theta - b \cos \theta$, show that $y^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 0$.

22. If $y = \left(x + \sqrt{1+x^2}\right)^n$, then show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2y$.

23. Find whether the following function is differentiable at $x = 1$ and $x = 2$ or not:

$$f(x) = \begin{cases} x, & x < 1 \\ 2-x, & 1 \leq x \leq 2 \\ -2+3x-x^2, & x > 2 \end{cases}$$

[Ans: $f(x)$ is not differentiable at $x = 1$ and differentiable at $x = 2$]

24. If $y = x^3 \log\left(\frac{1}{x}\right)$, then prove that $x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3x^2 = 0$.

25. Examine the following function $f(x)$ for continuity at $x = 1$ and differentiability at $x = 2$.

$$f(x) = \begin{cases} 5x-4, & 0 < x < 1 \\ 4x^2-3x, & 1 < x < 2 \\ 3x+4, & x \geq 2 \end{cases}$$

Ans: Bonus

26. If $\frac{x}{x-y} = \log \frac{a}{x-y}$, then prove that $\frac{dy}{dx} = 2 - \frac{x}{y}$.

27. If $x = \alpha \sin 2t(1 + \cos 2t)$ and $y = \beta \cos 2t(1 - \cos 2t)$, show $\frac{dy}{dx} = \frac{\beta}{\alpha} \tan t$.

28. Find $\frac{d}{dx} \cos^{-1}\left(\frac{x-x^{-1}}{x+x^{-1}}\right)$.

Ans: $-\frac{2}{1+x^2}$

29. Find the derivative of the following function $f(x)$ w.r.t. x , at $x = 1$.

$$\cos^{-1}\left[\sin \sqrt{\frac{1+x}{2}}\right] + x^x$$

Ans: $\frac{3}{4}$

30. If $y = e^{ax} \cdot \cos bx$, then prove that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)y = 0$.

31. If $x^x + x^y + y^x = a^b$, then find $\frac{dy}{dx}$. [Ans: $-\left(\frac{x^x(1+\log x) + yx^{y-1} + y^x \log y}{x^y \log x + xy^{x-1}}\right)$]

32. If $x = a \sin 2t (1 + \cos 2t)$ and $y = b \cos 2t (1 - \cos 2t)$, then find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

[Ans: $\frac{b}{a}$]

33. If function $f(x) = |x - 3| + |x - 4|$, then show that $f(x)$ is not differentiable at $x = 3$ and $x = 4$.

34. If $y = x e^{-x^2}$, find $\frac{dy}{dx}$. [Ans: $x e^{-x^2} e^{-x^2} \left(\frac{1}{x} - 2x \log x \right)$]

35. If $\log \sqrt{x^2 + y^2} = \tan^{-1} \left(\frac{x}{y} \right)$, then show that $\frac{dy}{dx} = \frac{y - x}{y + x}$.

36. If $y = \sqrt{x+1} - \sqrt{x-1}$, prove that $(x^2 - 1) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - \frac{1}{4} y = 0$.

DANGAL

[4 marks each]

1. Show that the function $f(x) = |x-1| + |x+1|$, for all $x \in R$, is not differentiable at the points $x = -1$ and $x = 1$.

2. If $f(x) = \sqrt{x^2 + 1}$; $g(x) = \frac{x+1}{x^2+1}$ and $h(x) = 2x-3$, then find $f'[h'\{g'(x)\}]$.

[Ans: $\frac{2}{\sqrt{5}}$]

3. Let $f(x) = x - |x - x^2|$, $x \in [-1,1]$. Find the point of discontinuity, (if any), of this function on $[-1,1]$.

Ans: No point of discontinuity.

4. Find $\frac{dy}{dx}$ if $y = \sin^{-1} \left[\frac{6x - 4\sqrt{1-4x^2}}{5} \right]$. [Ans: $\frac{2}{\sqrt{1-4x^2}}$]

5. If $x^y = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$.

6. Prove that $\frac{d}{dx} \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) \right] = \sqrt{a^2 - x^2}$.

7. Differentiate $\sin^{-1} \left[\frac{2^{x+1} \cdot 3^x}{1 + (36)^x} \right]$ with respect to x . [Ans: $\left[\frac{2^{x+1} \cdot 3^x}{1 + (36)^x} \right] \cdot \log 6$]

8. If $y = \log \left(\frac{x}{a+bx} \right)^x$, prove that $x^3 \frac{d^2y}{dx^2} = \left(x \frac{dy}{dx} - y \right)^2$.

9. Find $\frac{d}{dx} \cos^{-1} \left(\frac{x-x^{-1}}{x+x^{-1}} \right)$. [Ans: $-\frac{2}{1+x^2}$]

10. Find the derivative of the following function $f(x)$ w.r.t. x , at $x = 1$.

$$\cos^{-1} \left[\sin \sqrt{\frac{1+x}{2}} \right] + x^x$$

[Ans: $\frac{3}{4}$]

11. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, prove that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$.

12. Show that the function f given by $f(x) = \begin{cases} \frac{e^{1/x} - 1}{e^{1/x} + 1}, & \text{if } x \neq 0 \\ -1, & \text{if } x = 0 \end{cases}$ is discontinuous at

$x = 0$.

13. Find k , if $f(x) = \begin{cases} k \sin \frac{\pi}{2}(x+1), & x \leq 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$ is continuous at $x = 0$.

14. Find the values of a and b , if the function f defined by

$$f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1 \\ bx + 2, & x > 1 \end{cases} \text{ is differentiable at } x = 1.$$

[Ans: $a = 3, b = 5$]

15. Find the values of p and q , for which $f(x) = \begin{cases} \frac{1 - \sin^3 x}{3 \cos^2 x}, & \text{if } x < \frac{\pi}{2} \\ p, & \text{if } x = \frac{\pi}{2} \\ \frac{q(1 - \sin x)}{(\pi - 2x)^2}, & \text{if } x > \frac{\pi}{2} \end{cases}$

is continuous at $x = \frac{\pi}{2}$.

[Ans: $p = \frac{1}{2}, q = 4$]