2. Motion in a Straight Line

Course: **NCERT based JEE Main, NEET** Class: 11 CBSE/GSEB Subject: **PHYSICS**

NCERT Simplified

Additional topics* ***2025-26*

2.1 Introduction

Motion is change in position of an object with time.**Rectilinear motion** is motion of objects along a straight line.

Here the objects in motion shall be treated as point objects.This approximation is valid so far as the size of the object is much smaller than the distance it moves in a reasonable duration of time.The size of objects can be neglected and they can be considered as **point-like objects** without much error.

Kinematics is the study of motion without going into the causes of motion.

*Position, Pathlength and Displacement

Motion is change in position of an object with time. In order to specify position, a reference point and a set of axes is chosen.

Consider a rectangular coordinate system consisting of three mutually perpendicular axes, labelled $X-$, $Y-$ and $Z-$ axes.The point of intersection of these three axes is called origin $(O)$and serves as the reference point.The coordinates (x, y, z) of an object describe the position of the object with respect to this coordinate system.To measure time, a clock is positioned in this system. This coordinate system along with a clock constitutes a **frame of reference.**

If one or more coordinates of an object change with time, the object is said to be in motion. Otherwise the object is said to be at rest with respect to this frame of reference.

The choice of a set of axes in a frame of reference depends upon the situation.For describing motion in two/three dimensions, a set of two/three axes is needed.

Description of an event depends on the frame of reference chosen for the description. For example, when a car is moving on a road, the car is said to be moving with respect to a frame of reference attached to the ground.But with respect to a frame of reference attached with a person sitting in the car, the car is at rest.

To describe motion along a staight line, one axis is chosen, say $X - axis$ so that it coincides with the path of the object. The position of the object is measured with reference to a conveniently chosen origin, here O, as shown in figure below.

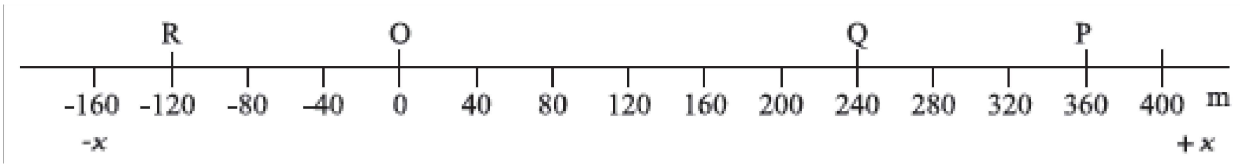


Figure 1

Positions to the right of O are taken as positive and to the left of O as negative.Following this convention, the position coordinates of point P and Q in figure are $+360\,m$ and $+240\,m$.Similarly, the position coordinate of point R is $-120\,m$.

⇒ **Pathlength**

Pathlength is defined as the total length of the path traversed by an object.

Consider the motion of a car along a straight line. The $X - axis$ is chosen such that it coincides with the path of the car's motion and origin of the axis as the point from where the car started moving, i.e. the car was at $x = 0$ at $t = 0$ as shown in figure 1.Let P, Q and R represent the positons of the car at different instants of time.Consider two cases of motion. In the first case, the car moves from O to P. Then the distance moved by the car is $OP = +360\,m$.This distance is called the path length traversed by the car.In the second case, the car moves from O to P and then moves back from P to Q.During this course of motion, the path length traversed is

$$OP + PQ = +360\,m + (+120\,m) = +480\,m$$

Pathlength is a **scalar quantity- a quantity that has a magnitude only and no direction**.

⇒ **Displacement**

Displacement is the change in position.

Let x_1 and x_2 be the positions of an object at time t_1 and t_2.Then its displacement is denoted by Δx, in time $\Delta t = (t_2 - t_1)$ is given by the difference between the final and initial positions: $\Delta x = x_2 - x_1$

(Greek letter delta Δ is used to denote a change in a quantity)

If $x_2 > x_1$, Δx is positive; and if $x_2 < x_1$, Δx is negative.

Displacement is a vector quantity as it has both magnitude and direction.

Motion along a straight line is also called rectilinear moton.In one-dimensional motion, there are only two directions (backward and forward, upward and downward) in which an object can move, and these two directions can easily be specified by $+$ and $-$ signs. For example, displacement of the car in moving from O to P is:

$$\Delta x = x_2 - x_1 = (+360\,m) - 0\,m = +360\,m$$

The displacement has a magnitude of $360\,m$ and is directed in the positive x direction as indicated by the $+$ sign.Similarly, the displacement of the car from P to Q is $240\,m - 360\,m = -120\,m$.**The negative sign indicates the direction of displacement**.Thus, **it is not necessary to use vector notation for discussing motion of objects in one-dimension**.

The magnitude of displacement may or may not be equal to the path length traversed by an object.

For example, for motion of the car from O to P, the path length is $+360\,m$ and the displacement is $+360\,m$. In this case, the magnitude of displacement ($360\,m$) is equal to the path length ($360\,m$).

Now, consider the motion of the car from O to P and back to Q.

In this case, the path length $= (+360\,m) + (120\,m) = +480\,m$

Displacement $(+240\,m) - (0\,m) = +240\,m$

Thus, the magnitude of displacement $(240\,m)$ is not equal to the path length $(480\,m)$.

The magnitude of the displacement for a course of motion may be zero but the corresponding path length is not zero. For example,it the car starts from O, goes to P and then returns to O, the final position coincides with the initial position and the displacement is zero. However, the path length of this journey is

$$OP + PO = 360\,m + 360\,m = 720\,m.$$

$\Rightarrow$ Motion of an object can be represented by a position-time graph. For motion along a straight line, say $X-$axis, only $x-$coordinate varies with time and $x-t$ graph is obtained.Consider an **object to be stationary**, e.g. a car standing still at $40\,m$.

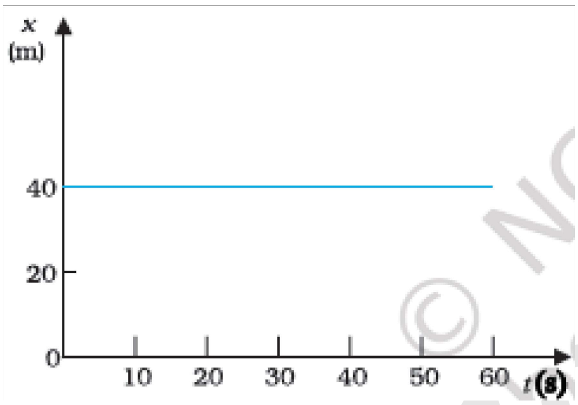


Figure 2

The **position-time graph is a straight line parallel to the time axis** as shown in figure 2 .

$\Rightarrow$ If an object moving along the straight line covers **equal distances in equal intervals of time**, it is said to be in **uniform motion** along a straight line.

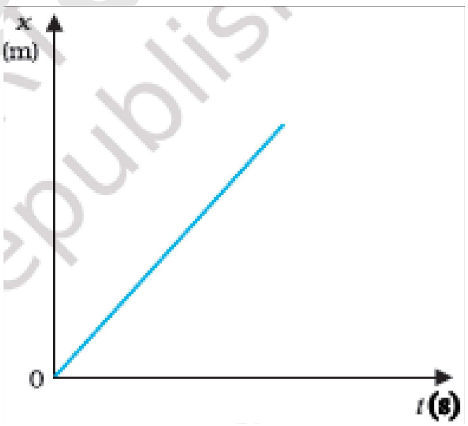
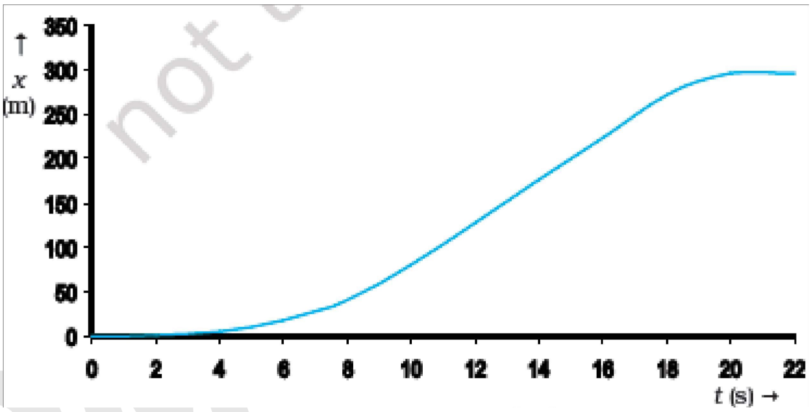


Figure 3

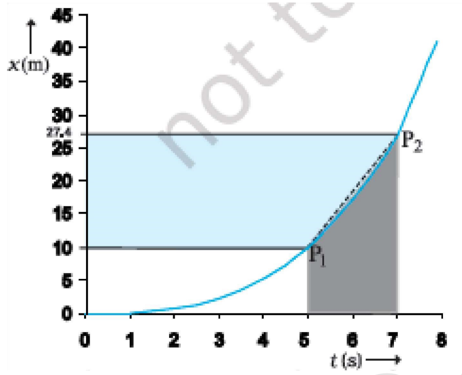
⇒ Consider the motion of a car that starts from rest at time $t = 0\,s$ from the origin O and pick up speed till $t = 10\,s$ and thereafter moves with uniform speed till $t = 18\,s$. Then the brakes are applied and the car stops at $t = 20\,s$ and $x = 296\,m$. The position-time graph for this case is shown in figure 1.

Figure 1



The portion of the $x - t$ graph between $t = 0\,s$ and $t = 8\,s$ is blown up and shown in figure 2 below:

Figure 2



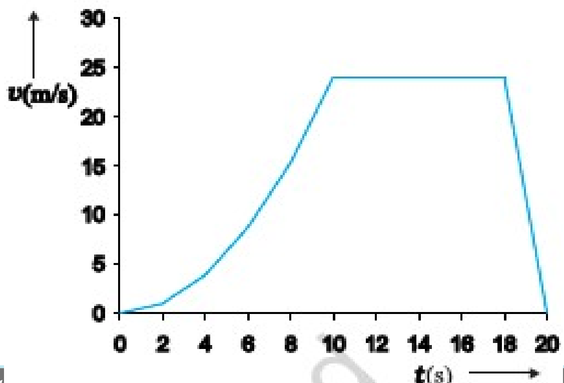
As seen from the plot, the average velocity of the car between time $t = 5\,s$ and $t = 7\,s$ is:

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{(27.4 - 10)\,m}{(7 - 5)\,s} = 8.7\,ms^{-1}$$

Geometrically, this is the slope of the straight line P_1P_2 connecting the initial position P_1 to the final position P_2 as shown in figure 2.

For this case, the variation of velocity with time is found to be as shown in figure 3.

Figure 3



During the period $t = 10\,s$ to $t = 18\,s$, the velocity is constant.Between period $t = 18\,s$ to $t = 20\,s$, it is uniformly decreasing and during the period $t = 0\,s$ to $t = 10\,s$, it is increasing.

Note that **for uniform motion, velocity is the same as the average velocity at all instants**.

On plot of velocity versus time, the **average acceleration is the slope of the straight line** connecting the points corresponding to (t_2, v_2) and (t_1, v_1) .Figure 4 is the acceleration versus time graph. Here the acceleration is nonuniform over the period $0\,s$ to $10\,s$. It is zero between $10\,s$ and $18\,s$ and is constant with value $-12\,m/s^2$ between $18\,s$ and $20\,s$.**When the acceleration is uniform, it equals the average acceleration over that period**.

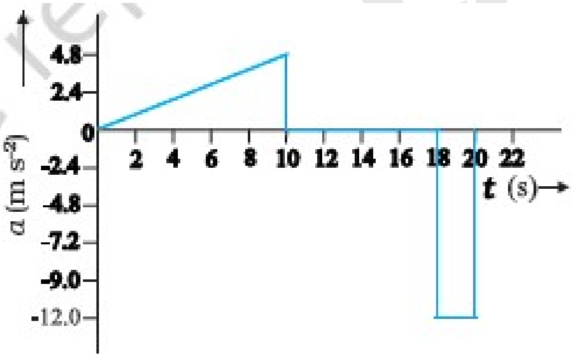


Figure 4

The average acceleration for velocity-time graph shown in figure 4 for different time intervals are:

$$0\,s - 10\,s: \quad \bar{a} = \frac{(24-0)\,ms^{-1}}{(10\text{-}0)\,s} = 2.4\,ms^{-2}$$

$$10\,s - 18\,s: \quad \bar{a} = \frac{(24-24)\,ms^{-1}}{(18\text{-}10)\,s} = 0\,ms^{-2}$$

$$18\,s - 20\,s: \quad \bar{a} = \frac{(0-24)\,ms^{-1}}{(20\text{-}18)\,s} = -12\,ms^{-2}$$

* Average Velocity and Average Speed

When an object is in motion, its position changes with time. Average velocity gives idea about how fast the positon is changing with time and in what direction.Average velocity is defined as the change in position or displacement (Δx) divided by the time intervals (Δt), in which the displacement occurs.

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

where x_2 and x_1 are the positions of the object at time t_2 and t_1 , respectively.Here the **bar** over the symbol for velocity is a standard notation used to indicate **an average quantity**. The **SI unit for velocity is** m/s **or** ms^{-1}, although $km\,h^{-1}$ is used in many everyday applications.

Average velocity is a vector quantity.Though for motion in a straight line, the directional aspect of the vector can be taken care of by $+$ and $-$ signs and the vector notation for velocity is not to be used in this chapter.

The average velocity can be positive or negative depending upon the sign of the displacement.It is zero if the displacement is zero.

Figure 1 shows $x-t$ graph for an object moving with positive velocity.

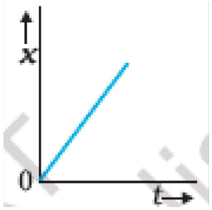


Figure 1

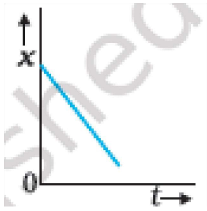


Figure 2

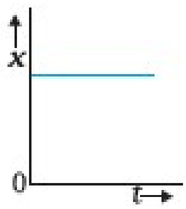


Figure 3

Figure 2 shows $x-t$ graph for an object moving with negative velocity.

Figure 3 shows $x-t$ graph for an object at rest.

The rate of motion over the actual path is described by average speed. **Average speed is defined as the total path length travelled devided by the total time interval during which the motion has taken place**.

$$Average\ speed = \frac{Total\ path\ length}{Total\ time\ interval}$$

Average speed has the same unit as that of velocity. But it does not tell us in what direction an object is moving. Thus it is always positive in contrast to the average velocity which can be positive or negative. **If the motion of an object is along a straight line and in the same direction, the magnitude of displacement is equal to the total path length**. In that case, the magnitude of average velocity is equal to the average speed.

PRACTICE QUESTIONS

Q. 1. A man walks on a straight road from his home to a market $2.5\ km$ away with a speed of $5\ kmh^{-1}$.

Finding the market closed, he instantly turns and walks back home with a speed of $7.5\ kmh^{-1}$. What is the

(a) magnitude of average velocity, and (b) average speed of the man

over the interval of time (i) 0 to 30 min (ii) 0 to 40 min (iii) 0 to 50 min **[NCERT]**

Q. 2. Draw the position time graph of an object moving with zero acceleration. **[1 mark]**

Q. 3. A particle moves along a circular path of radius R. What is the distance and displacement of the particle after one complete revolution. **[2 marks]**

Q. 4. Distinguish between speed and velocity. **[2 marks]**

QUESTIONS from Competitive Exams

1. An object moves with speed v_1, v_2 and v_3 along a line segment AB, BC and CD respectively as shown in figure. Where $AB = BC$ and $AD = 3AB$, then average speed of the object will be:

(A) $\frac{3v_1v_2v_3}{(v_1v_2+v_2v_3+v_3v_1)}$ (B) $\frac{(v_1+v_2+v_3)}{3v_1v_2v_3}$ (C) $\frac{v_1v_2v_3}{3(v_1v_2+v_2v_3+v_3v_1)}$ (D) $\frac{(v_1+v_2+v_3)}{3}$ **[JEE Main2023]**

2. A car travels a distance of $'x'$ with speed v_1 and then same distance $'x'$ with speed v_2 in the same direction. The average speed of the car is:

(A) $\frac{v_1v_2}{2(v_1+v_2)}$ (B) $\frac{v_1+v_2}{2}$ (C) $\frac{2v_1v_2}{v_1+v_2}$ (D) $\frac{2x}{v_1+v_2}$ **[JEE Main 2023]**

3. A car runs at a constant speed on a circular track of radius 100m taking 62.8 seconds for every circular lap. The average velocity and average speed for each circular lap respectively is

(A) 0,0 (B) 0, 10 m/s (C) 10 m/s, 10 m/s (D) 10 m/s, 0 **[AIPMT 2006]**

4. A car moves from X to Y with a uniform speed v_u and returns to Y with a uniform speed v_d. The average speed for this round trip is:

(A) $\frac{v_u + v_d}{2}$ (B) $\frac{2v_d v_u}{v_d + v_u}$ (C) $\sqrt{v_u v_d}$ (D) $\frac{v_d v_u}{v_d + v_u}$ **[AIPMT 2007]**

5. A particle covers half its total distance with speed v_1 and rest half with speed v_2. Its average speed during the complete journey is

(A) $\frac{v_1 + v_2}{2}$ (B) $\frac{v_1 v_2}{v_1 + v_2}$ (C) $\frac{2v_1 v_2}{v_1 + v_2}$ (D) $\frac{v_1^2 v_2^2}{v_1^2 + v_2^2}$ **[AIPMT 2011 Mains]**

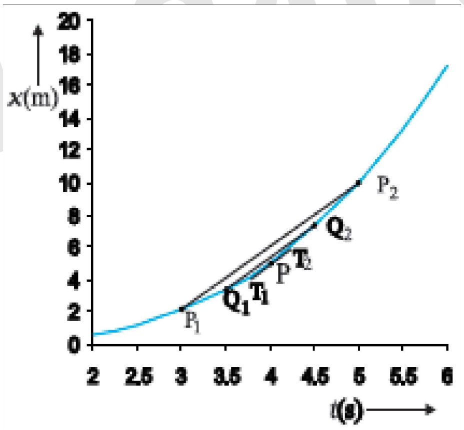
2.2 Instantaneous Velocity and Speed

The velocity at an instant or instantaneous velocity v at an instant t is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small. In other words, $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

The average velocity gives idea about how fast an object has been moving over a given time interval and instantaneous velocity gives idea about how fast it moves at different instants of time during that interval.

where the symbol $\lim_{\Delta t \to 0}$ stands for the operation of taking limit as $\Delta t \to 0$ of the quantitiy on its right. In the language of calculus, the quantity on the right hand side of above equation is the differential coefficient of x with respect to t and is denoted by $\frac{dx}{dt}$. $\frac{dx}{dt}$ **is the rate of change of position with respect to time, at that instant**.

Consider the graph shown below. Let us take $\Delta t = 2\,s$ centred at $t = 4\,s$.



Then, by the definition of the average velocity, the slope of the line P_1P_2 gives the value of average velocity over the interval $3\,s$ to $5\,s$. When the value of Δt is decreased from $2\,s$ to $1\,s$, then line P_1P_2 becomes Q_1Q_2 and its slope gives the value of the average velocity over the interval $3.5\ s$ to $4.5\ s$. In the limit $\Delta t \to 0$, the line P_1P_2 becomes tangent to the position-time curve at the point P and the velocity at $t = 4\ s$ is given by the slope of the tangent at that point. It is difficult to show this process graphically.

By using differential calculus, instantaneous velocity $v = \frac{dx}{dt}$.

For the graph shown above, $x = 0.08\,t^3$

$$v = \frac{d\left(0.08x^3\right)}{dt}$$
$$= (0.08)3\,x^2$$

Instantaneous velocity at $t = 4\ s$,

$$v = (0.08)3\,(4)^2$$
$$= (0.08)3\,(16)$$
$$= 3.24\ m/s$$

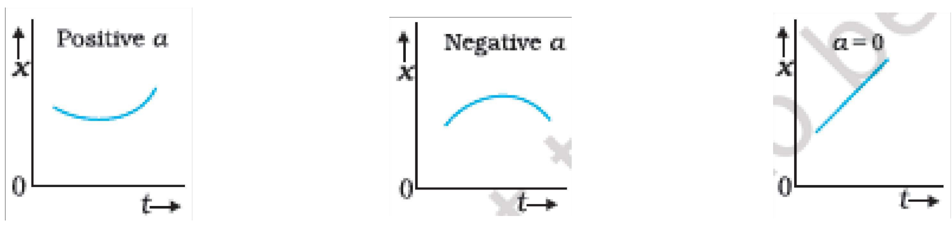
It should be noted that:
Instantaneous speed or simply speed is the magnitude of velocity.
Average speed over a finite interval of time is greater or equal to the magnitude of the average velocity.
Instantaneous speed at an instant is equal to the magnitude of the instantaneous velocity at that instant.

2.3 ACCELERATION

Acceleration is the rate of change of velocity with time.
The **average acceleration** $\bar{a}$ **over a time interval is defined as the change of velocity divided by the time interval**: $\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$

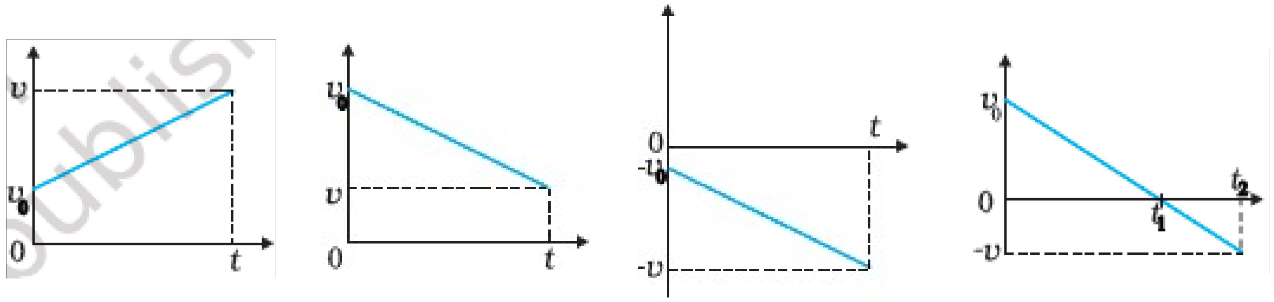
Instantaneous acceleration is $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

The acceleration at an instant is the slope of the tangent to the $v - t$ **curve at that instant**.
Since velocity is a quantity having both magnitude and direction, a change in velocity may involve either or both of these factors. **Acceleration may result from a change in speed (magnitude), a change in direction or changes in both**. Acceleration can be positive, negative or zero.
⇒ **Position-time graphs** for motion with positive, negative and zero acceleration are shown in figure below.



Note that the graph **curves upward for positive acceleration**; **downward for negative acceleration** and it is **a straight line for zero acceleration**.

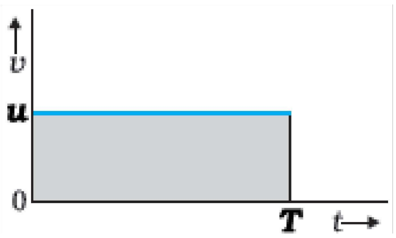
⇒ **Velocity-time** graph for motion with constant acceleration for the following cases is shown below:



(a) **(b)** **(c)** **(d)**

(a) An object is moving in a positive direction with a positive acceleration
(b) An object is moving in a positive direction with a negative acceleration
(c) An object is moving in a negative direction with a negative acceleration
(d) An object is moving in a positive direction till time t_1 and then turns back with the same negative acceleration.

⇒ An interesting feature of a **velocity-time graph for any moving object is that the area under the curve represents the displacement over a given time interval.** A $v-t$ graph of an object moving with constant velocity u is shown below:



The $v-t$ curve is a straight line parallel to the time axis and the area under it between $t=0$ and $t=T$ is the area of the rectangle of height u and base T.

Displacement in this time interval is $Area = u \times T = uT$

Note that the $x-t$, $v-t$ and $a-t$ graphs shown in several figures in this chapter have **sharp kinks** at some points implying that the **functions are not differentiable at these points**. In any realistic situation, the functions will be differentiable at all points and the graphs will be smooth.

This means **acceleration and velocity cannot change values abruptly at an instant**. **Changes are always continuous**.

PRACTICE QUESTIONS

Q.1. The position x of a body moving along a straight line at time t is given by $x = \left(3t^2 - 5t + 2\right) m$. Find:

(a) Velocity at $t = 2\,s$.

(b) Acceleration at $t = 2\,s$.

(c) Draw the corresponding velocity-time $v-t$ and acceleration-time $a-t$ graphs.

Q.2. Which physical quantity do the following represent

(a) slope of $x-t$ graph (b) slope of $v-t$ graph

(c) Area of $a-t$ graph (d) Area of $v-t$ graph

Q.3. The area under the acceleration-time graph gives .... for the given time interval.

(A) Speed (B) Velocity (C) Change in velocity (D) Distance travelled

IMPORTANT FORMULAE

1 - 25: Differentiation:

1. $\frac{d(u+v)}{dx}=\frac{du}{dx}+\frac{dv}{dx}$

2. $\frac{d(u-v)}{dx}=\frac{du}{dx}-\frac{dv}{dx}$

3. $\frac{d(u.v)}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$

4. $\frac{d\left(\frac{u}{v}\right)}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

5. $\frac{d(constant)}{dx}=0$

6. $\frac{d(u^n)}{dx}=nu^{n-1}.\frac{du}{dx}$ **OR** $\frac{d(x^n)}{dx}=nx^{n-1}$

7. $\frac{d(a^u)}{dx}=a^u.\log_e a.\frac{du}{dx}, a>0$

8. $\frac{d(e^u)}{dx}=e^u.\frac{du}{dx}$

9. $\frac{d(\sin u)}{dx}=\cos u.\frac{du}{dx}$

10. $\frac{d(\cos u)}{dx}=-\sin u.\frac{du}{dx}$

11. $\frac{d(\tan u)}{dx}=\sec^2 u.\frac{du}{dx}$

12. $\frac{d(\sec u)}{dx}=\sec u.\tan u.\frac{du}{dx}$

13. $\frac{d(\cot u)}{dx}=-\operatorname{cosec}^2 u.\frac{du}{dx}$

14. $\frac{d(\operatorname{cosec} u)}{dx}=-\operatorname{cosec} u.\cot u.\frac{du}{dx}$

15. $\frac{d(\sin^{-1}u)}{dx}=\frac{1}{\sqrt{1-u^2}}.\frac{du}{dx}$

16. $\frac{d(\cos^{-1}u)}{dx}=\frac{-1}{\sqrt{1-u^2}}.\frac{du}{dx}$

17. $\frac{d(\tan^{-1}u)}{dx}=\frac{1}{1+u^2}.\frac{du}{dx}$

18. $\frac{d(\cot^{-1}u)}{dx}=\frac{-1}{1+u^2}.\frac{du}{dx}$

19. $\frac{d(\sec^{-1}u)}{dx}=\frac{1}{|u|\sqrt{u^2-1}}.\frac{du}{dx}$

20. $\frac{d(\operatorname{cosec}^{-1}u)}{dx}=\frac{-1}{|u|\sqrt{u^2-1}}.\frac{du}{dx}$

21. $\frac{d(\log u)}{dx}=\frac{1}{u}.\frac{du}{dx}$

22-25: **Following formulae needs special attention**:

BASE	POWER	DIFFERENTIATION
22. Variable	Constant	$\frac{d(u^n)}{dx} = nu^{n-1}\frac{du}{dx}$
23. Variable	Variable	$\frac{dx^x}{dx} = x^x(1+\log x)$
24. Constant	Constant	$\frac{d(cons\tan t)}{dx} = 0$
25. Constant	Variable	$\frac{d(a^u)}{dx} = a^u \log_e a \frac{du}{dx}, a > 0$

26. ***Integration:*** $\int_a^b x^n .dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b$

QUESTIONS from Competitive Exams

1. The relation between time t and distance x is $t = ax^2 + bx$, where a and b are constants. The acceleration is:

(A) $-2av^2$ (B) $2av^2$ (C) $-2av^3$ (D) $2bv^3$ **[AIEEE 2005]**

2. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as

(A) $t^{1/2}$ (B) t^3 (C) t^2 (D) t **[AIEEE 2006]**

3. A particle moves along a straight line OX. At a time t (in seconds) the distance x (in metres) of the particle from O is given by $x = 40 + 12t - t^3$. How long would the particle travel before coming to rest:

(A) 24 m (B) 40 m (C) 56 m (D) 16 m **[AIPMT 2006]**

4. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is

(A) $v_0 + 2g + 3f$ (B) $v_0 + \frac{g}{2} + \frac{f}{3}$ (C) $v_0 + g + f$ (D) $v_0 + \frac{g}{2} + f$ **[AIEEE 2007]**

5. A particle moving along x-axis has acceleration f, at time t, given by $f = f_0\left(1 - \frac{t}{T}\right)$, where f_0 and T are constants. The particle at $t = 0$ has zero velocity. In the time interval between $t = 0$ and the instant when $f = 0$, the particle's velocity (v_x) is:

(A) $\frac{1}{2}f_0T$ (B) f_0T (C) $\frac{1}{2}f_0T^2$ (D) f_0T^2 **[AIPMT 2007]**

6. The position x of a particle with respect to time t along x-axis is given by $x = 9t^2 - t^3$, where x is in metres and t in seconds. What will be the position of this particle when it achieves maximum speed along the $+x$ direction ?
(A) 24 m (B) 32 m (C) 54m (D) 81m **[AIPMT 2007]**

7. A particle moves a distance x in time t according to equation $x = (t+5)^{-1}$. The acceleration of particle is proportional to
(A) $(velocity)^{3/2}$ (B) $(distance)^2$ (C) $(distance)^{-2}$ (D) $(velocity)^{2/3}$ **[Pre AIPMT 2010]**

8. An object moving with a speed of 6.25 m/s is decelerated at a rate given by $\frac{dv}{dt} = -2.5\sqrt{v}$, where v is instantaneous speed. The time taken by the object to come to the rest, would be
(A) 2 s (B) 4 s (C) 8 s (D) 1 s **[AIEEE 2011]**

9. The motion of a particle along a straight line is described by the equation $x = 8 + 12t - t^3$, where x is in meter and t in second. The retardation of a particle when velocity of the particle is zero, is
(A) 6 ms^{-2} (B) 12 ms^{-2} (C) 24 ms^{-2} (D) zero **[Pre AIPMT 2012]**

10. A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to: $v(x) = bx^{-2n}$ where b and n are constants and x is the position of the particle. The acceleration of the particle as function of x is give by:
(A) $-2nb^2x^{-4n-1}$ (B) $-2b^2x^{-2n+1}$ (C) $-2nb^2e^{-4n+1}$ (D) $-2nb^2x^{-2n-1}$ **[NEET 2015]**

11. If the velocity of a particle is $v = At + Bt^2$, where A and B are constants, then the distance travelled by it between $1s$ and $2s$ is
(A) $\frac{A}{2} + \frac{B}{3}$ (B) $\frac{3}{2}A + B$ (C) $3A + 7B$ (D) $\frac{3}{2}A + \frac{7}{3}B$ **[NEET 2016]**

12. Two cars P and Q start from a point at the same time in a straight line and their positions are represented by $x_P(t) = at + bt^2$ and $x_Q(t) = ft - t^2$. At what time do the cars have the same velocity ?
(A) $\frac{a+f}{2(1+b)}$ (B) $\frac{f-a}{2(1+b)}$ (C) $\frac{a-f}{1+b}$ (D) $\frac{a+f}{2(b-1)}$ **[NEET 2016 Retest]**

13. In some appropriate units, time (t) and position (x) relation of a moving particle is given by $t = x^2 + x$. The acceleration of the particle is:
(A) $+\frac{2}{(x+1)^3}$ (B) $+\frac{2}{(2x+1)}$ (C) $-\frac{2}{(x+2)^3}$ (D) $-\frac{2}{(2x+1)^3}$ **[NEET 2025]**

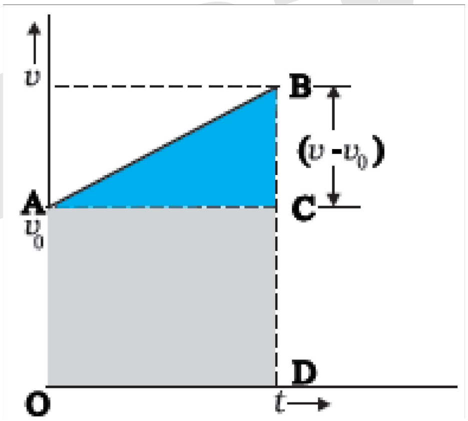
2.4 Kinematics Equations for Uniformly Accelerated Motion

For uniformly accelerated motion, some simple equations that relate displacement (x), time taken (t), initial velocity (v_0), final velocity (v) and acceleration (a) can be derived.

The relation between final velocity (v) and initial velocity (v_0) of an object moving with uniform acceleration (a) is given by

$$v = v_0 + at \qquad \text{....................(1)}$$

This relation is graphically represented in figure below:



The area under this $v-t$ curve is:

Area between instants 0 and t = Area of triangle ABC + Area of rectangle $OACD$.

$$= \frac{1}{2}(v - v_0)t + v_0 t$$

The **area under $v-t$ curve represents the displacement**.

The displacement x of the object is: $x = \frac{1}{2}(v - v_0)t + v_0 t$ ....(2)

As $v - v_0 = at$, $x = \frac{1}{2}at^2 + v_0 t$

or $x = v_0 t + \frac{1}{2}at^2$ ....(3)

From equation (2),

$$x = \frac{1}{2}vt - \frac{1}{2}v_0 t + v_0 t$$

$$= \frac{1}{2}vt + \frac{1}{2}v_0 t$$

$$= \frac{v + v_0}{2}t = \bar{v}\,t$$

where $\bar{v} = \frac{v + v_0}{2}$ (constant acceleration only)

Thus the object has undergone displacement x with an average velocity equal to the arithmetic average of the initial and final velocities.

From equation (1), $t = \frac{(v - v_0)}{a}$ and $x = \frac{v + v_0}{2} t$,

Thus $$x = \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right) = \frac{v^2 - {v_0}^2}{2a}$$

$$v^2 = {v_0}^2 + 2ax \qquad ....(3)$$

Three important equations are:

$$v = v_0 + at$$

$$x = v_0 t + \frac{1}{2} a t^2$$

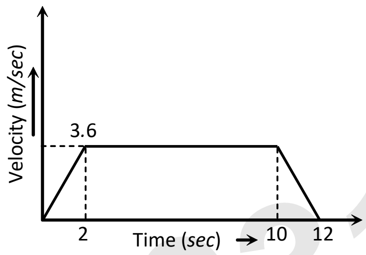
$$v^2 = {v_0}^2 + 2ax$$

PRACTICE QUESTIONS

Q. 1. What is the distance travelled by a car in its 3^{rd} second if it moves from rest with a uniform acceleration of $6\ m/s^2$?

(A) $20\,m$ (B) $35\ m$ (C) $15\ m$ (D) $40\ m$ **[1 mark]**

Q. 2. The velocity-time graph for the motion of a particle is given below. Draw the acceleration-time graph.



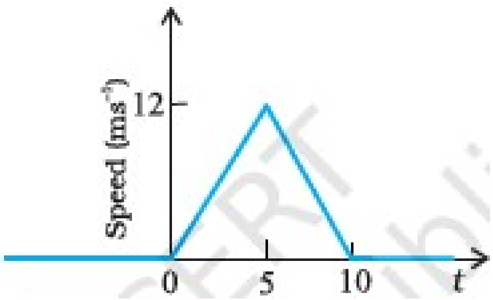
[2 marks]

Q. 3 Obtain equations of motion for constant acceleration using method of calculus. **[3 marks]**

Q. 4 Derive the following equations of motion for uniformly accelerated motion from velocity-time graph:

(a) $v = v_0 + at$ (b) $x = v_0 t + \frac{1}{2} a t^2$ (c) $v^2 = {v_0}^2 + 2ax$ **[3 marks]**

Q. 5 The speed-time graph of a particle moving along a fixed direction is shown in figure.



Obtain the distance traversed by the particle between (a) $t = 0\,s$ to $t = 10\,s$, (b) $t = 2\,s$ to $t = 6\,s$.

[Ans: (a) $60\,m$ **(b)** $36\,m$**]**

Q. 6 A car moving along a straight highway with speed of 126 kmh^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop ?

[Ans: $3.06\ m/s^2$, $11.4\ s$ **]** **[NCERT]**

....**QUESTIONS from Competitive Exams**..

1 . Speeds of two identical cars are u and $4u$ at a specific instant. The ratio of the respective distances in which the two cars are stopped from that instant is
(A) 1 : 1 (B) 1 : 4 (C) 1 : 8 (D) 1:16 **[AIEEE 2002]**

2. A car moving with a speed of 50 km/hr, can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is:
(A) 12 m (B) 18 m (C) 24 m (D) 6 m **[AIEEE 2003]**

3. An automobile travelling with a speed of 60 km/h, can brake to stop within a distance of 20 m. If the car is going twice as fast ie. 120 km/h, the stopping distance will be
(A) 20 m (B) 40 m (C) 60 m (D) 80 m **[AIEEE 2004]**

4. A ball is released from the top of a tower of height h metres. It takes T seconds to reach the ground. What is the position of ball in $\frac{T}{3}$ seconds ?
(A) $\frac{h}{9}$ metre from the ground (B) $\frac{7h}{9}$ metre from the ground
(C) $\frac{8h}{9}$ metre from the ground (D) $\frac{17h}{18}$ metre from the ground **[AIEEE 2004]**

5. A car starting from rest, accelerates at the rate f through a distance s, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance travelled is $15s$, then
(A) $s = ft$ (B) $s = \frac{1}{6}ft^2$ (C) $s = \frac{1}{72}ft^2$ (D) $s = \frac{1}{4}ft^2$ **[AIEEE 2005]**

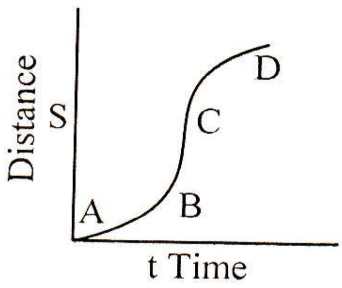
6. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 m/s^2. He reaches the ground with a speed of 3 m/s. At what height, did he bail out?
(A) 91 m (B)182 m (C) 293 m (D) 111 m **[AIEEE 2005]**

7. Two bodies, A(of mass 1kg) and B(of mass 3kg), are dropped from heights of 16 m and 25m respectively. The ratio of the time taken by them to reach the ground is:
(A) 5/4 (B) 12/5 (C) 5/12 (D) 4/5 **[AIPMT 2006]**

8. A particle moves in a straight line with a constant acceleration. It changes its velocity from 10 ms^{-1} to 20 ms^{-1} while passing through a distance 135 m in t second. The value of t is
(A) 12 (B) 9 (C) 10 (D) 1.8 **[Pre AIPMT 2008]**

9. The distance travelled by a particle starting from rest and moving with an acceleration $\frac{4}{3}ms^{-2}$, in the third second is
(A) 10/3 m (B) 19/3 m (C) 6m (D) 4m **[Pre AIPMT 2008]**

10. A particle shows distance-time curve as given in this figure. The maximum instantaneous velocity of the particle is around the point :



(A) D (B) A (C) B (D) C **[Pre AIPMT 2008]**

11. A particle starts its motion from rest under the action of a constant force. If the distance covered in first 10 seconds is S_1 and that covered in the first 20 seconds is S_2 then :

(A) $S_2 = S_1$ (B) $S_2 = 2S_1$ (C) $S_2 = 3S_1$ (D) $S_2 = 4S_1$ **[Pre AIPMT 2009]**

12. A bus is moving with a speed of $10 ms^{-1}$ on a straight road. A scooterist wishes to over take the bus in 100s. If the bus is at a distance of 1 km from the scooterist, with what speed should the scooterist chase the bus ?

(A) $10 ms^{-1}$ (B) $20 ms^{-1}$ (C) $40 ms^{-1}$ (D) $25 ms^{-1}$ **[Pre AIPMT 2009]**

13. A ball is dropped from a high rise platform at t = 0 starting from rest. After 6 second, another ball is thrown downwards from the same platform with speed v. The two balls meet at $t = 18$ sec. What is the value of v. (Take $g = 10 m/s^2$)

(A) 75 m/s (B) 55 m/s (C) 40 m/s (D) 60 m/s **[Pre AIPMT 2010]**

14. A boy standing at the top of a tower of 20m height drops a stone. Assuming $g = 10 m/s^2$, the velocity with which it hits the ground:

(A) 5 m/s (B) 10 m/s (C) 20 m/s (D) 40 m/s **[Pre AIPMT 2011]**

15. A stone falls freely under gravity. It covers distances h_1, h_2 and h_3 in the first 5 seconds, the next 5seconds and the next 5 seconds respectively. The relation between h_1, h_2 and h_3 is

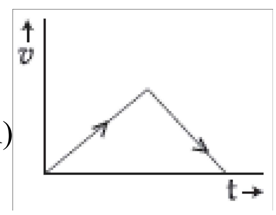
(A) $h_1 = 2h_2 = 3h_3$ (B) $h_1 = \frac{h_2}{3} = \frac{h_3}{5}$ (C) $h_2 = 3h_1$ and $h_3 = 3h_2$ (D) $h_1 = h_2 = h_3$ **[NEET 2013]**

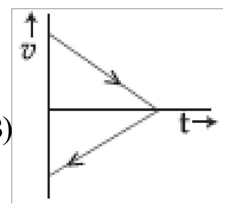
16. From a tower of height H, a parrticle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path.
The relation between H, u and n is:

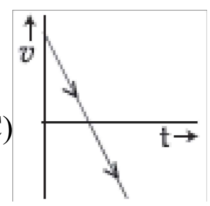
(A) $2gH = n^2u^2$ (B) $gH = (n-2)^2u^2$ (C) $2gH = nu^2(n-2)$ (D) $gH = (n-2)u^2$

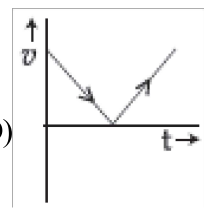
[JEE Main 2014]

17. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time ?

(A)


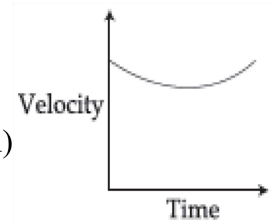
(B)


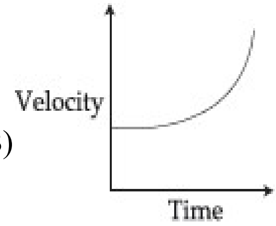
(C)


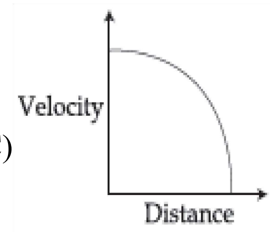
(D)


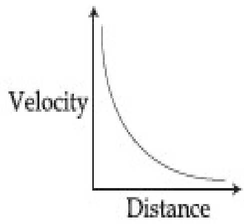
[JEE Main 2017]

18. Which graph corresponds to an object moving with a constant negative acceleration and a positive velocity ?

(A) 

(B) 

(C) 

(D) 

[JEE Main 2017]

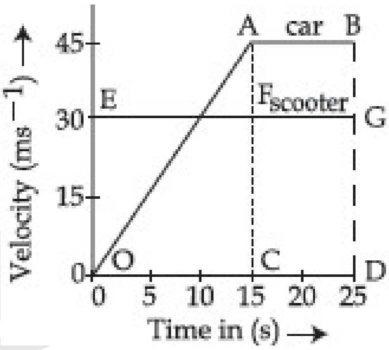
19. An automobile, travelling at $40\ km/h$, can be stopped at a distance of $40\ m$ by applying brakes. If the same automobile is travelling at $80\ km/h$, the minimum stopping distance, in metres, is (assume no skidding):

(A) 75 m (B) 100 m (C) 150 m (D)160 m **[JEE Main 2018]**

19. A bullet from a gu is fired on a rectangular wooden block with velocity u. When bullet travels $24\ cm$ through the block along its length horizontally, velocity of bullet becomes $\frac{u}{3}$. Then it further penetrates into the block in the same direction before coming to rest exactly at the other end of the block. The total length of the block is:

(A) $27\ cm$ (B) $24\ cm$ (C) $28\ cm$ (D) $30\ cm$ **[NEET 2023]**

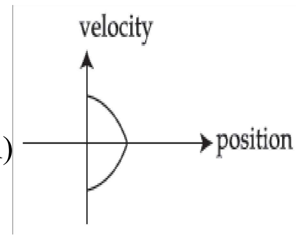
20. The velocity-time graphs of a car and a scooter are shown in the figure. (i) The difference between the distance travelled by the car and the scooter in $15\ s$ and (ii) the time at which the car will catch up with the scooter are, respectively.

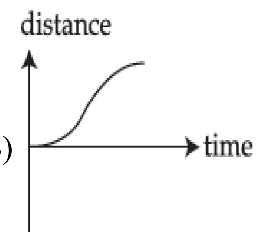


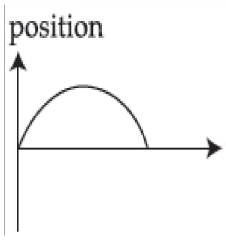
(A)112.5 m and 22.5 s (B)337.5 m and 25 s (C)112.5 m and 15 s (D)225.5 m and 10 s

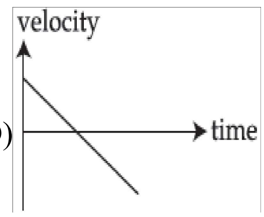
[JEE Main 2018 Online]

21. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

(A) 

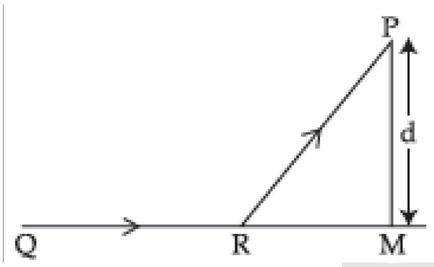
(B) 

(B) 

(D) 

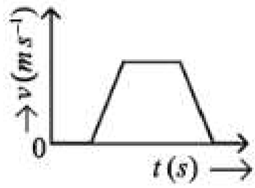
[JEE Main 2018]

22*. A man in a car at location Q on a straight highway is moving with speed v. He decides to reach a point P in a field at a distance d from the highway (point M) as shown in the figure. Speed of the car in the field is half to that on the highway. What should be the distance RM, so that the time taken to reach P is minimum ?

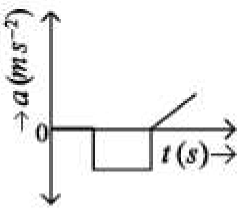
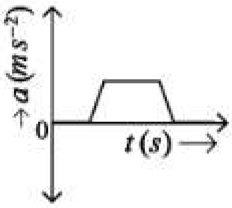
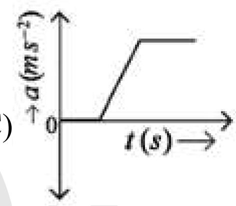
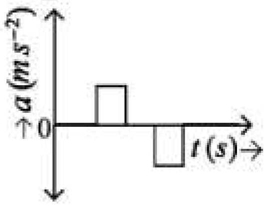


(A) d (B) $\frac{d}{\sqrt{2}}$ (C) $\frac{d}{2}$ (D) $\frac{d}{\sqrt{3}}$ **[JEE Main 2018]**

23. The velocity (v)-time (t) plot of the motion of a body is shown below:



The acceleration (a)- time (t) graph that best suits this motion is:

(A)  (B)  (C)  (D) 

[NEET 2024]