

ASSIGNMENT

CHAPTER 3: Inverse Trigonometric Functions

CLASS: 12 NCERT

SUBJECT: MATHEMATICS

Assignment 2.1

Basic Concepts

:Points to remember:

* The domains and ranges (principal value branches) of inverse trigonometric functions:

FUNCTIONS	DOMAIN	RANGE (Principal Value Branches)
$y = \sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \operatorname{cosec}^{-1} x$	$R - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$y = \tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1} x$	R	$(0, \pi)$

.....**Questions from CBSE Question paper.....**

[1 mark Questions]

1. Find the value of $\tan^{-1} \sqrt{3} - \cot^{-1}(-\sqrt{3})$. Ans: $-\frac{\pi}{2}$
[All India 2018]
2. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$, $xy < 1$, then write the value of $x + y + xy$. Ans: 1
[All India 2014]
3. Write the principal value of $\cos^{-1}[\cos(680^\circ)]$. Ans: 40°
[Delhi 2014C]
4. Write the principal value of $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$. Ans: $-\frac{\pi}{4}$
[All India 2014C]
5. Find the value of $\cot\left(\frac{\pi}{2} - 2 \cot^{-1} \sqrt{3}\right)$. Ans: $\sqrt{3}$
[All India 2014C]
6. Write the principal value of $\left[\cos^{-1} \frac{\sqrt{3}}{2} + \cos^{-1}\left(-\frac{1}{2}\right)\right]$. Ans: $\frac{5\pi}{6}$
[Delhi 2013C]
7. Write the value of $\tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}\left(\frac{a-b}{a+b}\right)$. Ans: $\frac{\pi}{4}$
[Delhi 2013C]
8. Write the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$. Ans: $\frac{11\pi}{12}$
[Delhi 2013]
9. Write the value of $\tan\left(2 \tan^{-1} \frac{1}{5}\right)$. Ans: $\frac{5}{12}$
[Delhi 2013]
10. Write the value of $\tan^{-1}\left[2 \sin\left(2 \cos^{-1} \frac{\sqrt{3}}{2}\right)\right]$. Ans: $\frac{\pi}{3}$
[All India 2013]

11. Write the principal value of $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$. **Ans:** $-\frac{\pi}{2}$
[All India 2013]
12. Write the value of $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$. **Ans:** $\frac{2\pi}{3}$
[Delhi 2012]
13. Write the value of $\cot(\tan^{-1} a + \cot^{-1} a)$. **Ans:** 0
[Delhi 2012]
14. Find the principal value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$. **Ans:** $-\frac{\pi}{3}$
[All India 2012]
15. Using the principal values, write the value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$. **Ans:** $\frac{2\pi}{3}$
[All India 2012C]
16. Write the values of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$. **Ans:** 1
[Delhi 2011]
17. Write the value of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$. **Ans:** $-\frac{\pi}{4}$
[Delhi 2011]
18. Write the value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$. **Ans:** $\frac{5\pi}{6}$
[Delhi 2011, 2009; All India 2009]
19. What is the principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$. **Ans:** π
[All India 2011, 2008, 2009C]
20. What is the principal value of $\tan^{-1}(-1)$? **Ans:** $-\frac{\pi}{4}$
[Foreign 2011, 2008C]

21. Using the principal values, write the value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.
Ans: $-\frac{\pi}{3}$
[All India 2011C]
22. Write the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$.
Ans: $-\frac{\pi}{6}$
[Delhi 2011C]
23. Write the principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.
Ans: $\frac{\pi}{3}$
[Delhi 2010]
24. What is the principal value of $\sec^{-1}(-2)$?
Ans: $\frac{2\pi}{3}$
[All India 2010]
25. What is the domain of the function $\sin^{-1} x$.
Ans: $[-1,1]$
[Foreign 2010]
26. Using the principal values, find the value of $\cos^{-1}\left(\cos \frac{13\pi}{6}\right)$.
Ans: $\frac{\pi}{6}$
[All India 2010C]
27. If $\tan^{-1}(\sqrt{3}) + \cot^{-1} x = \frac{\pi}{2}$, then find the value of x .
Ans: $\sqrt{3}$
[All India 2010C]
28. Write the principal value of $\sin^{-1}\left(\sin \frac{3\pi}{5}\right)$.
Ans: $\frac{2\pi}{5}$
[Delhi 2009]
29. Using the principal values, evaluate $\tan^{-1}(1) + \sin^{-1}\left(-\frac{1}{2}\right)$.
Ans: $\frac{\pi}{12}$
[Delhi 2009C]
30. Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.
Ans: $\frac{\pi}{6}$
[All India 2008C]

Assignment 2.2

Properties of Inverse Trigonometric Functions

Important Formulae:

1-A. $y = \sin^{-1} x \Rightarrow x = \sin y$

2-A. $\sin(\sin^{-1} x) = x$

3-A. $\sin^{-1} \frac{1}{x} = \operatorname{cosec}^{-1} x$

3-B. $\cos^{-1} \frac{1}{x} = \sec^{-1} x$

3-C. $\tan^{-1} \frac{1}{x} = \cot^{-1} x$

5-A. $\sin^{-1}(-x) = -\sin^{-1} x$

5-B. $\tan^{-1}(-x) = -\tan^{-1} x$

5-C. $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$

7-A. $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$

8-A. $2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}$

1-B. $x = \sin y \Rightarrow y = \sin^{-1} x$

2-B. $\sin^{-1}(\sin x) = x$

4-A. $\cos^{-1}(-x) = \pi - \cos^{-1} x$

4-B. $\cot^{-1}(-x) = \pi - \cot^{-1} x$

4-C. $\sec^{-1}(-x) = \pi - \sec^{-1} x$

6-A. $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

6-B. $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$

6-C. $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$

7-B. $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x-y}{1+xy} \right)$

8-B. $2 \tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2} = \cos^{-1} \frac{1-x^2}{1+x^2}$

.....Questions from CBSE Question paper.....

[2 marks Questions]

1. Prove that $3 \sin^{-1} x = \sin^{-1}(3x - 4x^3)$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ [2018]

[4 marks Questions]

2. If $\tan^{-1}\left(\frac{x-3}{x-4}\right) + \tan^{-1}\left(\frac{x+3}{x+4}\right) = \frac{\pi}{4}$, then find the value of x .

Ans: $\sqrt{\frac{17}{2}}$ [2017]

3. Solve for x : $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}3x$. [2016]

Ans: $\pm \frac{1}{2}$

4. Prove that $\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right) - \tan^{-1}\left(\frac{4x}{1-4x^2}\right) = \tan^{-1}2x$; $|2x| < \frac{1}{\sqrt{3}}$ [2016]

5. Evaluate $\tan\left\{2 \tan^{-1}\left(\frac{1}{5}\right) + \frac{\pi}{4}\right\}$. [All India 2015]

Ans: $\frac{17}{7}$

6. If $\tan^{-1}\left(\frac{x-2}{x-4}\right) + \tan^{-1}\left(\frac{x+2}{x+4}\right) = \frac{\pi}{4}$, find the value of x . [All India 2014]

Ans: $\pm\sqrt{2}$

7. Prove that $\cos^{-1}x + \cos^{-1}\left\{\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right\} = \frac{\pi}{3}$. [All India 2014C]

8. Solve for x : $\tan^{-1}x + 2 \cot^{-1}x = \frac{2\pi}{3}$. [All India 2014C]

Ans: $\sqrt{3}$

9. Prove that $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$. [Delhi 2013C]

10. Solve for x , $\tan^{-1} 3x + \tan^{-1} 2x = \frac{\pi}{4}$.

Ans: $\frac{1}{6}$

[Delhi 2013C, 2009; All India 2009C, 2008]

11. Find the value of the following

$$\tan \frac{1}{2} \left[\sin^{-1} \left(\frac{2x}{1+x^2} \right) + \cos^{-1} \left(\frac{1-y^2}{1+y^2} \right) \right], \text{ if } |x| < 1, y > 0 \text{ and } xy < 1.$$

Ans: $\frac{x+y}{1-xy}$

[Delhi 2013]

12. Prove that $\tan^{-1} \left(\frac{1}{2} \right) + \tan^{-1} \left(\frac{1}{5} \right) + \tan^{-1} \left(\frac{1}{8} \right) = \left(\frac{\pi}{4} \right)$.

[Delhi 2013; All India 2011, 2008C]

13. Show that $\tan \left(\frac{1}{2} \sin^{-1} \frac{3}{4} \right) = \frac{4-\sqrt{7}}{3}$.

14. Solve the following equation $\cos \left(\tan^{-1} x \right) = \sin \left(\cot^{-1} \frac{3}{4} \right)$.

Ans: $\frac{3}{4}$

[All India 2013]

15. Prove the following:

$$\cos \left(\sin^{-1} \frac{3}{5} + \cot^{-1} \frac{3}{2} \right) = \frac{6}{5\sqrt{13}}$$

[All India 2012]

16. Prove that $\sin^{-1} \left(\frac{8}{17} \right) + \sin^{-1} \left(\frac{3}{5} \right) = \cos^{-1} \left(\frac{36}{85} \right)$.

[All India 2014C, Delhi 2012, 2010C]

17. Prove that $\tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right) = \frac{\pi}{4} - \frac{x}{2}, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$.

[Delhi 2012]

18. Prove that $\cos^{-1} \left(\frac{4}{5} \right) + \cos^{-1} \left(\frac{12}{13} \right) = \cos^{-1} \left(\frac{33}{65} \right)$.

[All India 2012; Delhi 2010C, 2009]

19. Prove that $\sin^{-1} \left(\frac{63}{65} \right) = \sin^{-1} \left(\frac{5}{13} \right) + \cos^{-1} \left(\frac{3}{5} \right)$.

[Foreign 2012]

20. Solve for x , $2 \tan^{-1}(\sin x) = \tan^{-1}(2 \sec x), x \neq \frac{\pi}{2}$.

Ans: $\frac{\pi}{4}$

[Foreign 2012]

21. Prove the following $\cot^{-1} \left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right] = \frac{x}{2}; x \in \left(0, \frac{\pi}{4} \right)$.

[Delhi 2014C]

22. Find the value of $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$. Ans: $\frac{\pi}{4}$
[Delhi 2011]
23. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x, -\frac{1}{\sqrt{2}} \leq x \leq 1$. [All India 2014]
24. Prove that $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$. [All India 2011, 2008C]
25. Prove that $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$. [Foreign 2011]
26. Solve following equation for x , $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x, x > 0$ Ans: $\frac{1}{\sqrt{3}}$
[Foreign 2011C, 08C; All India 2010, 2009C, 2014C]
27. Prove that $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\tan^{-1}\frac{4}{3}$.
[All India 2011C]
28. Solve for x , $\cos\left(2\sin^{-1}x\right) = \frac{1}{9}, x > 0$. Ans: $\frac{2}{3}$
[All India 2011]
29. Prove that $2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31} = \frac{\pi}{4}$. [Delhi 2011C]
30. Solve for x , $\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}, -1 < x < 1$. Ans: $2 - \sqrt{3}$
[Delhi 2011C]
31. Prove that $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in (0, 1)$. [Delhi 2010]
32. Prove that $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$. [Delhi 2010]
33. Prove that $\tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$. [All India 2010]

34. Prove that $\cos \left[\tan^{-1} \left\{ \sin \left(\cot^{-1} x \right) \right\} \right] = \sqrt{\frac{1+x^2}{2+x^2}}$. [All India 2010]
35. Solve for x , $\cos^{-1} x + \sin^{-1} \left(\frac{x}{2} \right) = \frac{\pi}{6}$. **Ans:** 1
[All India 2010C]
36. Prove that $2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} = \frac{\pi}{4}$. [All India 2010C]
37. Solve for x , $\tan^{-1} \frac{x}{2} + \tan^{-1} \frac{x}{3} = \frac{\pi}{4}$, $\sqrt{6} > x > 0$ **Ans:** 1
[Delhi 2010C]
38. Solve for x , $\tan^{-1}(x+2) + \tan^{-1}(x-2) = \tan^{-1} \left(\frac{8}{79} \right)$, $x > 0$ **Ans:** $\frac{1}{4}$
[Delhi 2010C]
39. Solve for x , $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$. **Ans:** $\frac{\pi}{4}$
[Delhi 2014C, All India 2009]
40. Prove that $\sin^{-1} \left(\frac{4}{5} \right) + \sin^{-1} \left(\frac{5}{13} \right) + \sin^{-1} \left(\frac{16}{65} \right) = \frac{\pi}{2}$. [Delhi 2009]
41. Prove that $\tan^{-1} \left(\frac{3}{4} \right) + \tan^{-1} \left(\frac{3}{5} \right) - \tan^{-1} \left(\frac{8}{19} \right) = \frac{\pi}{4}$. [All India 2009C]
42. Prove that $\tan^{-1} \left(\frac{1}{3} \right) + \tan^{-1} \left(\frac{1}{5} \right) + \tan^{-1} \left(\frac{1}{7} \right) + \tan^{-1} \left(\frac{1}{8} \right) = \frac{\pi}{4}$.
[All India 2009C; Delhi 2008, 2008C]
43. Solve for x , $\tan^{-1} x + 2 \cot^{-1} x = \frac{2\pi}{3}$. **Ans:** $\sqrt{3}$
[Delhi 2009C]
44. Solve for x , $\tan^{-1} \left(\frac{x-1}{x-2} \right) + \tan^{-1} \left(\frac{x+1}{x+2} \right) = \frac{\pi}{4}$. **Ans:** $\pm \frac{1}{\sqrt{2}}$
[Delhi 2009C]
45. Solve for x , $\tan^{-1} \left(\frac{1+x}{1-x} \right) = \frac{\pi}{4} + \tan^{-1} x$, $0 < x < 1$. **Ans:** Many solutions.
[Delhi 2008C]

.....**REVISION SESSION # 1**.....

1. Evaluate $\tan\left\{2 \tan^{-1}\left(\frac{1}{5}\right) + \frac{\pi}{4}\right\}$.

Ans: $\frac{17}{7}$

[4 marks]

2. Solve for x : $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} \frac{8}{31}$

Ans: $\frac{1}{4}$

[4 marks]

3. Prove the following:

$$\cot^{-1}\left(\frac{xy+1}{x-y}\right) + \cot^{-1}\left(\frac{yz+1}{y-z}\right) + \cot^{-1}\left(\frac{zx+1}{z-x}\right) = 0. \quad (0 < xy, yz, zx < 1)$$

[4 marks]

4. Find the value of x , if $\sin[\cot^{-1}(x+1)] = \cos(\tan^{-1} x)$.

Ans: $-\frac{1}{2}$

[4 marks]

5. Prove the following:

$$2 \sin^{-1} \frac{3}{5} - \tan^{-1} \frac{17}{31} = \frac{\pi}{4}$$

[4 marks]

6. If $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$, prove that $\sin y = \tan^2\left(\frac{x}{2}\right)$.

[4 marks]

7. Solve for x : $\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = \tan^{-1}(-7)$

Ans: 2

[4 marks]

8. If $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$, then find x .

Ans: -1

[4 marks]

9. Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$.

Ans: $\frac{1}{6}, -1$

[4 marks]

10. Prove that $\tan^{-1} \frac{63}{16} = \sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5}$. [4 marks]

11. Prove that $2 \tan^{-1} \left(\frac{1}{2} \right) + \tan^{-1} \left(\frac{1}{7} \right) = \sin^{-1} \left(\frac{31}{25\sqrt{2}} \right)$. [4 marks]

12. Solve for x : $\tan^{-1} \left(\frac{1-x}{1+x} \right) = \frac{1}{2} \tan^{-1} x, x > 0$.

Ans: $\frac{1}{\sqrt{3}}$ [4 marks]

13. If $2 \tan^{-1}(\cos \theta) = \tan^{-1}(2 \operatorname{cosec} \theta), (\theta \neq 0)$, then find the value of θ .

Ans: $\frac{\pi}{4}$ [4 marks]

14. If $\tan^{-1} \left(\frac{1}{1+1.2} \right) + \tan^{-1} \left(\frac{1}{1+2.3} \right) + \dots + \tan^{-1} \left(\frac{1}{1+n.(n+1)} \right) = \tan^{-1} \theta$, then find the value of θ .

Ans: $\frac{n}{n+2}$ [4 marks]

15. Prove the following:

$$\sin \left[\tan^{-1} \left(\frac{1-x^2}{2x} \right) + \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) \right] = 1, \quad 0 < x < 1. \quad [4 \text{ marks }]$$

16. If $\tan^{-1} \left(\frac{x-5}{x-6} \right) + \tan^{-1} \left(\frac{x+5}{x+6} \right) = \frac{\pi}{4}$, then find the value of x .

Ans: $x = \pm \frac{7}{\sqrt{2}}$ [4 marks]

17. Prove that $2 \tan^{-1} \left(\sqrt{\frac{a-b}{a+b}} \tan \frac{x}{2} \right) = \cos^{-1} \left(\frac{a \cos x + b}{a + b \cos x} \right)$. [4 marks]

18. Solve the following for x :

$$\tan^{-1} \left(\frac{x-2}{x-3} \right) + \tan^{-1} \left(\frac{x+2}{x+3} \right) = \frac{\pi}{4}, \quad |x| < 1.$$

Ans: No solution [4 marks]

19. Solve the following for x :

$$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}.$$

Ans: 0

[4 marks]

20. Show that: $2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$

[4 marks]

21. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, $x, y, z > 0$, then find the value of $xy + yz + zx$.

Ans: 1

[4 marks]

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1. If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$, prove that $\frac{x^2}{a^2} - 2\frac{xy}{ab}\cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha$. [4 marks]

2. Prove that $\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right) - \tan^{-1}\left(\frac{4x}{1-4x^2}\right) = \tan^{-1}2x$; $|2x| < \frac{1}{\sqrt{3}}$ [4 marks]

3. Evaluate $\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right) + \frac{\pi}{4}\right\}$.

Ans: $\frac{17}{7}$

[4 marks]

4. Prove the following:

$$\cot^{-1}\left(\frac{xy+1}{x-y}\right) + \cot^{-1}\left(\frac{yz+1}{y-z}\right) + \cot^{-1}\left(\frac{zx+1}{z-x}\right) = 0. \quad (0 < xy, yz, zx < 1)$$

[4 marks]

5. If $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$, prove that $\sin y = \tan^2\left(\frac{x}{2}\right)$. [4 marks]