

Continuity and Differentiation

Course: JEE Main/GUJCET

Worksheet: 1

Subject: MATHS

1. Let $S = \left\{ t \in R : f(x) = |x - \pi| \left(e^{|x|} - 1 \right) \cdot \sin|x| \text{ is not differentiable at } t \right\}$. Then the set S is equal to:
 (A) φ (an empty set) (B) $\{0\}$ (C) $\{\pi\}$ (D) $\{0, \pi\}$ [JEE Main 2018]
2. If $x^2 + y^2 + \sin y = 4$, then the value of $\frac{d^2y}{dx^2}$ at the point $(-2, 0)$ is:
 (A) -34 (B) -32 (C) 4 (D) -2 [JEE Main Online 2018]
3. Let $S = \left\{ (\lambda, \mu) \in R \times R : f(t) = \left(|\lambda| e^{|t|} - \mu \right) \cdot \sin(2|t|), t \in R, \text{ is a differentiable function} \right\}$. Then S is a subset of:
 (A) $R \times [0, \infty)$ (B) $[0, \infty) \times R$ (C) $R \times (-\infty, 0)$ (D) $(-\infty, 0) \times R$ [JEE Main Online 2018]
4. If $f(x) = \sin^{-1} \left(\frac{2 \times 3^x}{1 + 9^x} \right)$, then $f' \left(-\frac{1}{2} \right)$ equals:
 (A) $-\sqrt{3} \log_e \sqrt{3}$ (B) $\sqrt{3} \log_e \sqrt{3}$ (C) $-\sqrt{3} \log_e 3$ (D) $\sqrt{3} \log_e 3$ [JEE Main Online 2018]
5. Let $f(x)$ be a polynomial of degree 4 having extreme values at $x = 1$ and $x = 2$. If $\lim_{x \rightarrow 0} \left(\frac{f(x)}{x^2} + 1 \right) = 3$ then $f(-1)$ is equal to:
 (A) $\frac{9}{2}$ (B) $\frac{5}{2}$ (C) $\frac{3}{2}$ (D) $\frac{1}{2}$ [JEE Main Online 2018]
6. If $x = \sqrt{2^{\cos^{-1} t}}$ and $y = \sqrt{2^{\sec^{-1} t}}$ ($|t| \geq 1$) then $\frac{dy}{dx}$ is equal to:
 (A) $\frac{y}{x}$ (B) $\frac{x}{y}$ (C) $-\frac{y}{x}$ (D) $-\frac{x}{y}$ [JEE Main Online 2018]
7. If for $x \in \left(0, \frac{1}{4} \right)$, the derivative of $\tan^{-1} \left(\frac{6x\sqrt{x}}{1-9x^3} \right)$ is $\sqrt{x} \cdot g(x)$, then $g(x)$ equals:
 (A) $\frac{3x\sqrt{x}}{1-9x^3}$ (B) $\frac{3x}{1-9x^3}$ (C) $\frac{3x}{1+9x^3}$ (D) $\frac{9}{1+9x^3}$ [JEE Main 2017]
8. If $y = \left[x + \sqrt{x^2 - 1} \right]^{15} + \left[x - \sqrt{x^2 - 1} \right]^{15}$, then $(x^2 - 1) \frac{d^2y}{dx^2} + x \frac{dy}{dx}$ is equal to:
 (A) $125y$ (B) $225y^2$ (C) $225y$ (D) $224y^2$ [JEE Main Online 2017]

9. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then :

- (A) g is not differentiable at $x = 0$ (B) $g'(0) = \cos(\log 2)$
(C) $g'(0) = -\cos(\log 2)$ (D) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

[JEE Main 2016]

10. If the function $f(x) = \begin{cases} -x, & x < 1 \\ a + \cos^{-1}(x+b), & 1 \leq x \leq 2 \end{cases}$ is differentiable at $x = 1$, then $\frac{a}{b}$ is equal to:

- (A) $-1 - \cos^{-1}(2)$ (B) $\frac{\pi - 2}{2}$ (C) $\frac{-\pi - 2}{2}$ (D) $\frac{\pi + 2}{2}$ [JEE Main Online 2016]

11. Let f be a polynomial function such that $f(3x) = f'(x) \cdot f''(x)$, for all $x \in R$. Then:

- (A) $f(2) - f'(2) + f''(2) = 10$ (B) $f''(2) - f(2) = 4$
(C) $f''(2) - f'(2) = 0$ (D) $f(2) + f'(2) = 28$

[JEE Main Online 2017]

12. If the function $g(x) = \begin{cases} k\sqrt{x+1}, & 0 \leq x \leq 3 \\ mx+2, & 3 < x \leq 5 \end{cases}$ is differentiable, then the value of $k + m$ is :

- (A) $\frac{16}{5}$ (B) $\frac{10}{3}$ (C) 4 (D) 2 [JEE Main 2015]

13. $\frac{d \log|x|^e}{dx}$

- (A) e^x (B) $\frac{1}{(\log x)^2}$ (C) $\frac{-1}{x(\log|x|)^2}$ (D) $\frac{1}{|x|}$ [GUJCET 2018]

14. $\frac{d}{dx} \tan^{-1}\left(\frac{1-x}{1+x}\right) =$

- (A) $\frac{-2}{1+x^2}$ (B) $\frac{-1}{1+x^2}$ (C) $\frac{2}{1+x^2}$ (D) $\frac{1}{1+x^2}$ [GUJCET 2018]

15. If $x = at^2$, $y = 2at$, then $\frac{d^2x}{dy^2} =$

- (A) $\frac{1}{2a}$ (B) $-2at^3$ (C) $\frac{-1}{2at^3}$ (D) $\frac{-1}{t^2}$ [GUJCET 2018]

16. $\frac{d}{dx} \left(\sqrt{3} \sin\left(2x + \frac{\pi}{3}\right) + \cos\left(2x + \frac{\pi}{3}\right) \right) =$ _____.

- (A) $4 \cos 2x$ (B) $-4 \sin 2x$ (C) $4 \sin 2x$ (D) $-4 \cos 2x$ [GUJCET 2017]

17. For the curve $f(x) = (x - 5)^2$, applying mean value theorem on $[4, 6]$ the tangent at _____ is parallel to the chord joining $A(4,1)$, $B(6,1)$.

- (A) $(4,6)$ (B) $\left(\frac{9}{2}, \frac{1}{4}\right)$ (C) $(0,5)$ (D) $(5,0)$ [GUJCET 2017]

18. Function $f(x) = \begin{cases} (\log_2 2x)^{\log_2 x}, & x \neq 1 \\ (k-1)^3; & x = 1 \end{cases}$ is continuous at $x = 1$, then $k =$ _____.

- (A) $e+1$ (B) $e^{1/3}$ (C) e^3 (D) $e-1$ [GUJCET 2017]